Notes on the function gsw_isopycnal_slope_ratio(SA, CT, p)

This function **gsw_isopycnal_slope_ratio**(SA,CT,p) evaluates the ratio r of the slope on the $S_\mathrm{A} - \Theta$ diagram of an isoline of potential density with reference pressure p_r to the slope of a potential density surface with reference pressure p, and is defined by (from section 3.17 of the TEOS-10 Manual, IOC *et al*. (2010))

$$r = \frac{\alpha^\Theta\left(S_\mathrm{A},\Theta,p\right)\big/\beta^\Theta\left(S_\mathrm{A},\Theta,p\right)}{\alpha^\Theta\left(S_\mathrm{A},\Theta,p_\mathrm{r}\right)\big/\beta^\Theta\left(S_\mathrm{A},\Theta,p_\mathrm{r}\right)} \ . \qquad (3.17.2)$$

This function, **gsw_isopycnal_slope_ratio**(SA,CT,p), uses the 75-term polynomial function expression for specific volume **gsw_specvol**(SA,CT,p). This 75-term polynomial expression for specific volume is discussed in Roquert *et al*. (2015) and in appendix A.30 and appendix K of the TEOS-10 Manual (IOC *et al*. (2010)).For dynamical oceanography we may take the 75-term polynomial expression for specific volume as essentially reflecting the full accuracy of TEOS-10.

References

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org

Roquet, F., G. Madec, T. J. McDougall and P. M. Barker, 2015: Accurate polynomial expressions for the density and specific volume of seawater using the TEOS-10 standard. Ocean Modelling, 90, pp. 29-43. http://dx.doi.org/10.1016/j.ocemod.2015.04.002

Here follows section 3.17 of the TEOS-10 Manual (IOC *et al*. (2010)).

3.17 Property gradients along potential density surfaces

The two-dimensional gradient of a scalar φ along a potential density surface $\nabla_\sigma \varphi$ is related to the corresponding gradient in the neutral tangent plane $\nabla_n \varphi$ by

$$\nabla_\sigma \varphi = \nabla_n \varphi + \frac{\varphi_z}{\Theta_z} \frac{R_\rho\left[r-1\right]}{\left[R_\rho - r\right]} \nabla_n \Theta \qquad (3.17.1)$$

(from McDougall (1987a)), where r is the ratio of the slope on the $S_\mathrm{A} - \Theta$ diagram of an isoline of potential density with reference pressure p_r to the slope of a potential density surface with reference pressure p, and is defined by

$$r = \frac{\alpha^\Theta\left(S_\mathrm{A},\Theta,p\right)\big/\beta^\Theta\left(S_\mathrm{A},\Theta,p\right)}{\alpha^\Theta\left(S_\mathrm{A},\Theta,p_\mathrm{r}\right)\big/\beta^\Theta\left(S_\mathrm{A},\Theta,p_\mathrm{r}\right)} \ . \qquad (3.17.2)$$

Substituting $\varphi = \Theta$ into (3.17.1) gives the following relation between the (parallel) isopycnal and epineutral gradients of Θ

$$\nabla_\sigma \Theta = \frac{r\left[R_\rho - 1\right]}{\left[R_\rho - r\right]} \nabla_n \Theta = G^\Theta \nabla_n \Theta \qquad (3.17.3)$$

where the "isopycnal temperature gradient ratio"

$$G^{\Theta} \;\equiv\; \frac{\left[R_{\rho}-1\right]}{\left[R_{\rho}/r-1\right]} \tag{3.17.4}$$

has been defined as a shorthand expression for future use. This ratio G^{Θ} is available in the GSW Toolbox from the algorithm **gsw_isopycnal_vs_ntp_CT_ratio**, while the ratio r of Eqn. (3.17.2) is available there as **gsw_isopycnal_slope_ratio**. Substituting $\varphi = S_{\mathrm{A}}$ into Eqn. (3.17.1) gives the following relation between the (parallel) isopycnal and epineutral gradients of S_{A}

$$\nabla_{\sigma} S_{\mathrm{A}} \;=\; \frac{\left[R_{\rho}-1\right]}{\left[R_{\rho}-r\right]}\,\nabla_{n} S_{\mathrm{A}} \;=\; \frac{G^{\Theta}}{r}\,\nabla_{n} S_{\mathrm{A}}. \tag{3.17.5}$$