

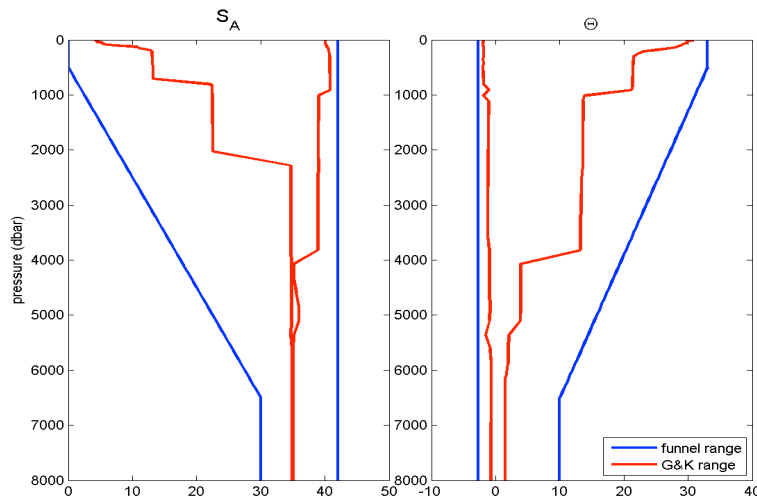
## Notes on the function gsw\_beta(SA,CT,p)

This function, `gsw_beta(SA,CT,p)`, evaluates the saline contraction coefficient with respect to Absolute Salinity at constant Conservative Temperature from the 48-term rational function expression for density of `gsw_rho(SA,CT,p)`. This 48-term rational function expression for density is discussed in McDougall and Barker (2013) and in appendix A.30 and appendix K of the TEOS-10 Manual (IOC *et al.* (2010)). The saline contraction coefficient  $\beta^\ominus$  is defined as

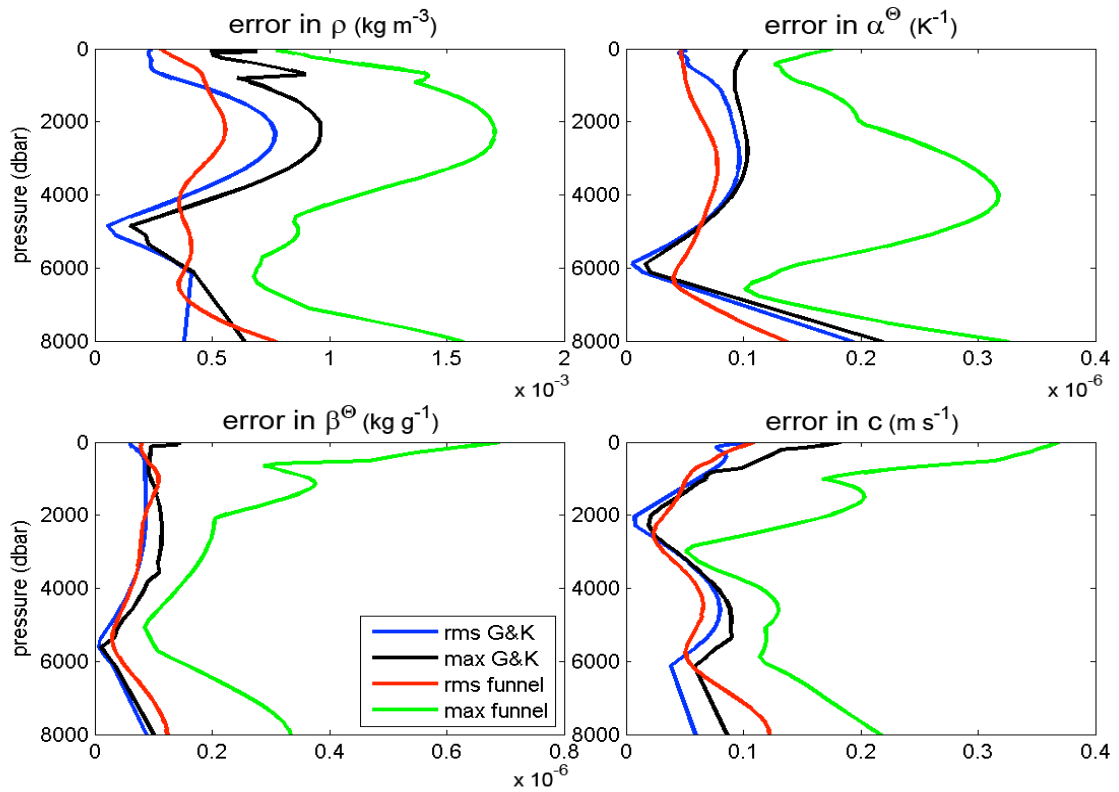
$$\beta^\ominus = \frac{1}{\rho} \left. \frac{\partial \rho}{\partial S_A} \right|_{\Theta, p}, \quad (1)$$

and when evaluated from the 48-term computationally-efficient expression for density, the rms error compared with the same saline contraction coefficient evaluated directly from the TEOS-10 Gibbs function is  $0.1 \times 10^{-6} \text{ kg g}^{-1}$  in the “oceanographic funnel” (see Figures 1 and 2 below). Compared with the mean value of  $\beta^\ominus$ , this error is small compared with the ratio of the uncertainty in the thermal expansion coefficient to its mean value, and so the error in  $\beta^\ominus$  is negligible. Hence we may take the saline contraction coefficient  $\beta^\ominus$  evaluated from `gsw_beta(SA,CT,p)` as essentially reflecting the full accuracy of TEOS-10. Exact values of  $\beta^\ominus$  may be evaluated using `gsw_beta_CT_exact(SA,CT,p)`.

Further comments on the 48-term rational function expression for density may be found in the Help file of `gsw_rho(SA,CT,p)`.



**Figure 1.** The ranges of Absolute Salinity and Conservative Temperature in the “oceanographic funnel” (the blue lines) in which the 48-term expression for density was fitted. The red lines shows the minimum and maximum values of Absolute Salinity and Conservative Temperature that occur in a hydrographic ocean atlas of the world ocean.



**Figure 2.** The errors in using the 48-term rational function expression for density, to evaluate density, the thermal expansion coefficient, the saline contraction coefficient and sound speed. The red and green lines are the r.m.s. and maximum errors for seawater in the “oceanographic funnel” (see Figure 1), while the blue and black lines are the r.m.s. and maximum errors for data in the world ocean.

## References

- McDougall T. J. and P. M. Barker, 2013: A computationally efficient 48-term expression for the density of seawater in terms of Conservative Temperature, and related properties of seawater. To be submitted to *Journal of Atmospheric and Ocean Technology*.
- IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from <http://www.TEOS-10.org>