Notes on the function gsw_isopycnal_slope_ratio(SA, CT, p)

This function **gsw_isopycnal_slope_ratio**(SA,CT,p) evaluates the ratio r of the slope on the $S_{\text{A}} - \Theta$ diagram of an isoline of potential density with reference pressure p_{r} to the slope of a potential density surface with reference pressure p, and is defined by (from section 3.17 of the TEOS-10 Manual, IOC *et al*. (2010))

$$r = \frac{\alpha^{\Theta}(S_{\text{A}},\Theta,p)/\beta^{\Theta}(S_{\text{A}},\Theta,p)}{\alpha^{\Theta}(S_{\text{A}},\Theta,p_{\text{r}})/\beta^{\Theta}(S_{\text{A}},\Theta,p_{\text{r}})}\,. \qquad (3.17.2)$$

This function, **gsw_isopycnal_slope_ratio**(SA,CT,p), uses the 48-term expression for density, $\hat{\rho}(S_{\text{A}},\Theta,p)$. This 48-term rational function expression for density is discussed in appendix A.30 and appendix K of the TEOS-10 Manual (IOC *et al*. (2010)). For dynamical oceanography we may take the 48-term rational function expression for density as essentially reflecting the full accuracy of TEOS-10.

References

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org

Here follows section 3.17 of the TEOS-10 Manual (IOC *et al*. (2010)).

3.17 Property gradients along potential density surfaces

The two-dimensional gradient of a scalar φ along a potential density surface $\nabla_{\sigma}\varphi$ is related to the corresponding gradient in the neutral tangent plane $\nabla_{n}\varphi$ by

$$\nabla_{\sigma}\varphi = \nabla_{n}\varphi + \frac{\varphi_z}{\Theta_z}\frac{R_{\rho}[r-1]}{[R_{\rho}-r]}\nabla_{n}\Theta \qquad (3.17.1)$$

(from McDougall (1987a)), where r is the ratio of the slope on the $S_{\text{A}} - \Theta$ diagram of an isoline of potential density with reference pressure p_{r} to the slope of a potential density surface with reference pressure p, and is defined by

$$r = \frac{\alpha^{\Theta}(S_{\text{A}},\Theta,p)/\beta^{\Theta}(S_{\text{A}},\Theta,p)}{\alpha^{\Theta}(S_{\text{A}},\Theta,p_{\text{r}})/\beta^{\Theta}(S_{\text{A}},\Theta,p_{\text{r}})}\,. \qquad (3.17.2)$$

Substituting $\varphi=\Theta$ into (3.17.1) gives the following relation between the (parallel) isopycnal and epineutral gradients of Θ

$$\nabla_{\sigma}\Theta = \frac{r[R_{\rho}-1]}{[R_{\rho}-r]}\nabla_{n}\Theta = G^{\Theta}\nabla_{n}\Theta \qquad (3.17.3)$$

where the "isopycnal temperature gradient ratio"

$$G^{\Theta} \equiv \frac{\left[R_\rho - 1\right]}{\left[R_\rho / r - 1\right]} \tag{3.17.4}$$

has been defined as a shorthand expression for future use. This ratio G^{Θ} is available in the GSW Toolbox from the algorithm **gsw_isopycnal_vs_ntp_CT_ratio**, while the ratio r of Eqn. (3.17.2) is available there as **gsw_isopycnal_slope_ratio**. Substituting $\varphi = S_\mathrm{A}$ into Eqn. (3.17.1) gives the following relation between the (parallel) isopycnal and epineutral gradients of S_A

$$\nabla_\sigma S_\mathrm{A} = \frac{\left[R_\rho - 1\right]}{\left[R_\rho - r\right]} \nabla_n S_\mathrm{A} = \frac{G^{\Theta}}{r} \nabla_n S_\mathrm{A}. \tag{3.17.5}$$