

## Notes on the function **gsw\_ntp\_pt\_vs\_CT\_ratio(SA, CT, p)**

This function **gsw\_ntp\_pt\_vs\_CT\_ratio(SA,CT,p)** evaluates the ratio of the parallel gradients of potential temperature and Conservative Temperature in a neutral tangent plane. This ratio is evaluated from the last part of Eqn. (A.14.5) of the TEOS-10 Manual (IOC *et al.* (2010)), namely

$$\frac{|\nabla_n \theta|}{|\nabla_n \Theta|} = \frac{(\alpha^\ominus / \beta^\ominus)}{(\alpha^\theta / \beta^\theta)} = \frac{T_b^\ominus}{T_b^\theta} = \hat{\theta}_\ominus + [\alpha^\ominus / \beta^\ominus] \hat{\theta}_{SA}, \quad (\text{A.14.5})$$

This ratio is shown in Figure A.14.1 of the TEOS-10 Manual at  $p = 0$  dbar (and this figure is reproduced below).

This function, **gsw\_ntp\_pt\_vs\_CT\_ratio(SA,CT,p)**, uses the 75-term polynomial expression for specific volume to evaluate  $\alpha^\ominus / \beta^\ominus$  in Eqn. (A.14.5). 75-term polynomial expression for specific volume is discussed in Roquet *et al.* (2015) and in appendix A.30 and appendix K of the TEOS-10 Manual (IOC *et al.* (2010)). For dynamical oceanography we may take the 75-term polynomial function expression for specific volume as essentially reflecting the full accuracy of TEOS-10.

### References

- IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from <http://www.TEOS-10.org>
- Roquet, F., G. Madec, T. J. McDougall and P. M. Barker, 2015: Accurate polynomial expressions for the density and specific volume of seawater using the TEOS-10 standard. *Ocean Modelling*, **90**, pp. 29-43. <http://dx.doi.org/10.1016/j.ocemod.2015.04.002>

Here follows appendix A.14 of the TEOS-10 Manual (IOC *et al.* (2010)).

### **A.14 Advective and diffusive “heat” fluxes**

In section 3.23 and appendices A.8 and A.13 the First Law of Thermodynamics is shown to be practically equivalent to the conservation equation (A.21.15) for Conservative Temperature  $\Theta$ . We have emphasized that this means that the advection of “heat” is very accurately given as the advection of  $c_p^0 \Theta$ . In this way  $c_p^0 \Theta$  can be regarded as the “heat content” per unit mass of seawater and the error involved with making this association is approximately 1% of the error in assuming that either  $c_p^0 \theta$  or  $c_p(S_A, \theta, 0 \text{ dbar}) \theta$  is the

“heat content” per unit mass of seawater (see also appendix A.21 for a discussion of this point).

The conservative form (A.21.15) implies that the turbulent diffusive flux of heat should be directed down the mean gradient of Conservative Temperature rather than down the mean gradient of potential temperature. In this appendix we quantify the difference between these mean temperature gradients.

Consider first the respective temperature gradients along the neutral tangent plane. From Eqn. (3.11.2) we find that

$$(\alpha^\theta/\beta^\theta)\nabla_n\theta = \nabla_n S_A = (\alpha^\Theta/\beta^\Theta)\nabla_n\Theta, \quad (\text{A.14.1})$$

so that the epineutral gradients of  $\theta$  and  $\Theta$  are related by the ratios of their respective thermal expansion and saline contraction coefficients, namely

$$\nabla_n\theta = \frac{(\alpha^\Theta/\beta^\Theta)}{(\alpha^\theta/\beta^\theta)}\nabla_n\Theta. \quad (\text{A.14.2})$$

This proportionality factor between the parallel two-dimensional vectors  $\nabla_n\theta$  and  $\nabla_n\Theta$  is readily calculated and illustrated graphically. Before doing so we note two other equivalent expressions for this proportionality factor.

The epineutral gradients of  $\theta$ ,  $\Theta$  and  $S_A$  are related by (using  $\theta = \hat{\theta}(S_A, \Theta)$ )

$$\nabla_n\theta = \hat{\theta}_\Theta \nabla_n\Theta + \hat{\theta}_{S_A} \nabla_n S_A, \quad (\text{A.14.3})$$

and using the neutral relationship  $\nabla_n S_A = (\alpha^\Theta/\beta^\Theta)\nabla_n\Theta$  we find

$$\nabla_n\theta = \left(\hat{\theta}_\Theta + \left[\alpha^\Theta/\beta^\Theta\right]\hat{\theta}_{S_A}\right)\nabla_n\Theta. \quad (\text{A.14.4})$$

Also, in section 3.13 we found that  $T_b^\theta \nabla_n\theta = T_b^\Theta \nabla_n\Theta$ , so that we find the expressions

$$\frac{|\nabla_n\theta|}{|\nabla_n\Theta|} = \frac{(\alpha^\Theta/\beta^\Theta)}{(\alpha^\theta/\beta^\theta)} = \frac{T_b^\Theta}{T_b^\theta} = \hat{\theta}_\Theta + \left[\alpha^\Theta/\beta^\Theta\right]\hat{\theta}_{S_A}, \quad (\text{A.14.5})$$

and it can be shown that  $\alpha^\theta/\alpha^\Theta = \hat{\theta}_\Theta$  and  $\beta^\theta/\beta^\Theta = \left(1 + \left[\alpha^\Theta/\beta^\Theta\right]\hat{\theta}_{S_A}/\hat{\theta}_\Theta\right)$ , that is,  $\beta^\theta = \beta^\Theta + \alpha^\Theta \hat{\theta}_{S_A}/\hat{\theta}_\Theta$ . The ratios  $\alpha^\theta/\alpha^\Theta$  and  $\beta^\theta/\beta^\Theta$  have been plotted by Graham and McDougall (2011); interestingly  $\alpha^\theta/\alpha^\Theta$  is approximately a linear function of  $S_A$  while  $\beta^\theta/\beta^\Theta$  is approximately a function of only  $\Theta$ . The partial derivatives  $\hat{\theta}_\Theta$  and  $\hat{\theta}_{S_A}$  in the last part of Eqn. (A.14.5) are both independent of pressure while  $\alpha^\theta/\beta^\theta$  is a function of pressure. The ratio, Eqn. (A.14.5), of the epineutral gradients of  $\theta$  and  $\Theta$  is shown in Figure A.14.1 at  $p = 0$ , indicating that the epineutral gradient of potential temperature is sometimes more than 1% different to that of Conservative Temperature. This ratio  $|\nabla_n\theta|/|\nabla_n\Theta|$  is only a weak function of pressure. This ratio,  $|\nabla_n\theta|/|\nabla_n\Theta|$  (i.e. Eqn. (A.14.5)), is available in the GSW Oceanographic Toolbox as function **gsw\_ntp\_pt\_vs\_CT\_ratio**.

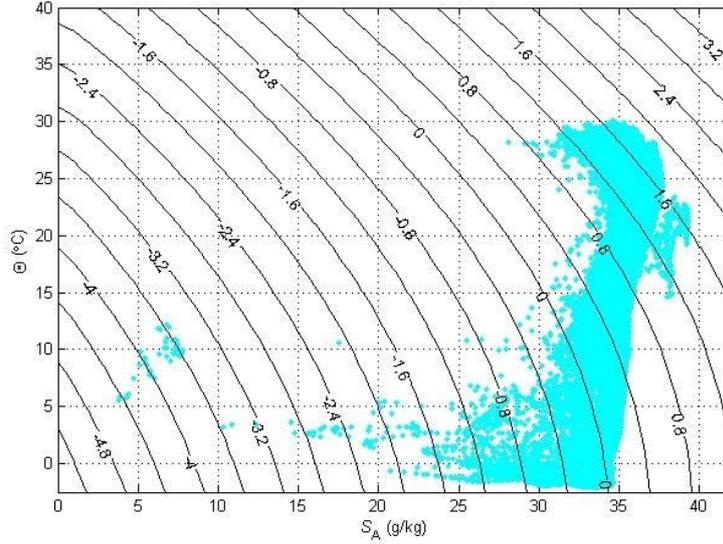
Similarly to Eqn. (A.14.3), the vertical gradients are related by

$$\theta_z = \hat{\theta}_\Theta \Theta_z + \hat{\theta}_{S_A} S_{A_z}, \quad (\text{A.14.6})$$

and using the definition, Eqn. (3.15.1), of the stability ratio we find that

$$\frac{\theta_z}{\Theta_z} = \hat{\theta}_\Theta + R_\rho^{-1} \left[\alpha^\Theta/\beta^\Theta\right] \hat{\theta}_{S_A}. \quad (\text{A.14.7})$$

For values of the stability ratio  $R_\rho$  close to unity, the ratio  $\theta_z/\Theta_z$  is close to the values of  $|\nabla_n\theta|/|\nabla_n\Theta|$  shown in Figure A.14.1.

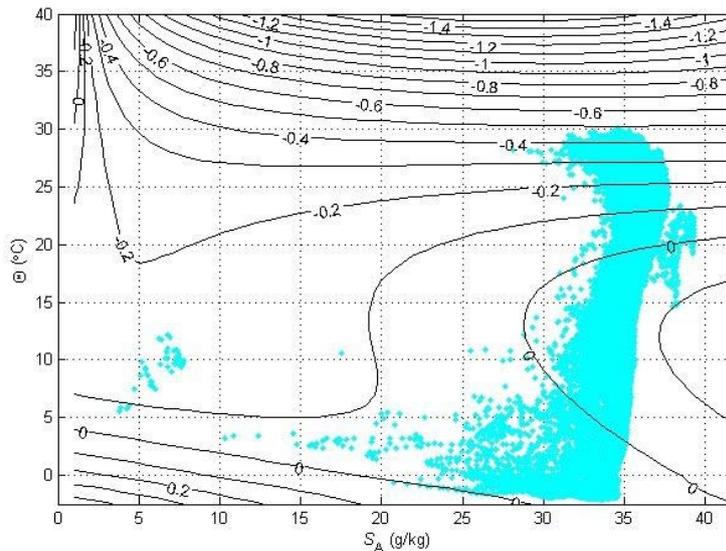


**Figure A.14.1.** Contours of  $(|\nabla_n \theta|/|\nabla_n \Theta| - 1) \times 100\%$  at  $p = 0$ , showing the percentage difference between the epineutral gradients of  $\theta$  and  $\Theta$ . The blue dots are from the ocean atlas of Gouretski and Koltermann (2004) at  $p = 0$ .

As noted in section 3.8 the dianeutral advection of thermobaricity is the same when quantified in terms of  $\theta$  as when done in terms of  $\Theta$ . The same is not true of the dianeutral velocity caused by cabbeling. The ratio of the cabbeling dianeutral velocity calculated using potential temperature to that using Conservative Temperature is given by  $(C_b^\theta \nabla_n \theta \cdot \nabla_n \theta) / (C_b^\Theta \nabla_n \Theta \cdot \nabla_n \Theta)$  (see section 3.9) which can be expressed as

$$\frac{C_b^\theta |\nabla_n \theta|^2}{C_b^\Theta |\nabla_n \Theta|^2} = \frac{C_b^\theta (\alpha^\Theta / \beta^\Theta)^2}{C_b^\Theta (\alpha^\theta / \beta^\theta)^2} = \frac{C_b^\theta (T_b^\Theta)^2}{C_b^\Theta (T_b^\theta)^2} = \frac{C_b^\theta}{C_b^\Theta} (\hat{\theta}_\Theta + [\alpha^\Theta / \beta^\Theta] \hat{\theta}_{S_A})^2, \quad (\text{A.14.8})$$

and this is contoured in Fig. A.14.2. While the ratio of Eqn. (A.14.8) is not exactly unity, it varies relatively little in the oceanographic range, indicating that the dianeutral advection due to cabbeling estimated using  $\theta$  or  $\Theta$  are within half a percent of each other at  $p = 0$ .



**Figure A.14.2.** Contours of the percentage difference of  $(C_b^\theta |\nabla_n \theta|^2) / (C_b^\Theta |\nabla_n \Theta|^2)$  from unity at  $p = 0$  dbar.