

## Notes on the function gsw\_steric\_height(SA,CT,p,p\_ref)

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The function **gsw\_geo\_strf\_dyn\_height**(SA,CT,p,p\_ref) delivers the dynamic height anomaly  $\Psi$  with units of  $\text{m}^2 \text{s}^{-2}$ , and the isobaric lateral gradient of this geostrophic streamfunction,  $\nabla_p \Psi$ , accurately represents the geostrophic velocity difference at pressure  $p$  relative to the (deep)  $p_{\text{ref}}$  pressure surface. That is, (from Eqn. (3.27.3) of the TEOS-10 Manual, IOC *et al.* (2010))

$$\mathbf{k} \times \nabla_p \Psi = f\mathbf{v} - f\mathbf{v}_{\text{ref}}. \quad (3.27.3)$$

Note that the use of the dynamic height anomaly  $\Psi$  to represent the difference between the geostrophic velocity at two different pressures is exact (under the assumption of geostrophy), and in particular, the dynamic balance of Eqn. (3.27.3) is totally consistent with the gravitational acceleration varying in three dimensional space. To belabour this point, Eqn. (3.27.3) holds without approximation or modification while the gravitational acceleration varies with latitude and pressure. This feature does not apply to steric height, as will now be explained.

It is not uncommon to seek an approximate geostrophic streamfunction which has dimensions of length (units of meters), and such an approximate geostrophic streamfunction is called “steric height”. Such a variable is convenient, for example, to compare with satellite-derived observations of the height of the sea surface with respect to the geoid.

Since the geopotential  $\Phi$  is defined by  $\nabla \Phi = g\mathbf{k}$  where  $\mathbf{k}$  is the unit vector pointing upwards in the vertical direction, and since the dynamic height anomaly  $\Psi$  is simply  $\Phi$  minus a function of pressure, it is clear that the isobaric derivatives of  $\Psi$  and  $\Phi$  are equal and that

$$\nabla_p \Psi = \nabla_p \Phi = g \nabla_p z. \quad (1)$$

One is then tempted to divide the dynamic height anomaly  $\Psi$  by the local value of the gravitational acceleration  $g = g(x, y, p)$  to obtain the “steric height”. The problem with this approach is that because the gravitational acceleration is a function of latitude

$$\nabla_p (\Psi/g) \quad (2)$$

is neither equal to  $\nabla_p z$  nor is it proportional to  $\nabla_p \Psi$  as it should be if  $(\Psi/g)$  were to be an exact geostrophic streamfunction in an isobaric surface. Hence it is not possible to form a “steric height” which is simultaneously (i) an exact geostrophic streamfunction in an isobaric surface, and (ii) is equal to the height of the isobaric surface above a geopotential surface (plus a spatially constant offset which is a function of latitude and longitude only).

Hence we here define the steric height to be the dynamic height anomaly  $\Psi$  divided by a fixed value representing the gravitational acceleration,  $g_0 = 9.7963 \text{ ms}^{-2}$ . This value of the gravitational acceleration is the areal average of the gravitational acceleration over the surface of the global ocean (see page 46 of Griffies (2004)). That is, in the present GSW function **gsw\_steric\_height**(SA,CT,p,p\_ref) we adopt the definition

$$\text{steric height} = \frac{\Psi}{9.7963 \text{ ms}^{-2}}. \quad (3)$$

From this definition it follows that the isobaric lateral gradient of steric height is proportional to an exact geostrophic streamfunction (see Eqn. (3.27.3) above), however this lateral gradient will not be exactly equal to  $\nabla_p z$ , the lateral gradient of the height of the isobaric surface above a geopotential surface.

In a typical use of this **gsw\_steric\_height**(SA,CT,p,p\_ref) function one inputs full ocean depth hydrographic profiles and chooses a “level of little motion” of say  $p_{\text{ref}} =$

2000 dbar, and one examines the output at  $p = 0$  dbar, and after subtracting a suitable constant offset value, one compares the results with the altimetric data at the sea surface.

### References

- Griffies, S. M., 2004: *Fundamentals of Ocean Climate Models*. Princeton, NJ: Princeton University Press, 518 pp + xxxiv.
- IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from <http://www.TEOS-10.org> See section 3.27 and appendix A.30.

Here follows section 3.27 of the TEOS-10 Manual (IOC *et al.* (2010)).

### 3.27 Dynamic height anomaly

The dynamic height anomaly  $\Psi$  with respect to the sea surface is given by

$$\Psi = - \int_{P_0}^P \hat{\delta}(S_A[p'], \Theta[p'], p') dP', \quad \text{where } \hat{\delta}(S_A, \Theta, p) = \hat{v}(S_A, \Theta, p) - \hat{v}(S_{SO}, 0^\circ\text{C}, p). \quad (3.27.1)$$

This is the geostrophic streamfunction for the flow at pressure  $P$  with respect to the flow at the sea surface and  $\hat{\delta}$  is the specific volume anomaly. Thus the two-dimensional gradient of  $\Psi$  in the  $P$  pressure surface is simply related to the difference between the horizontal geostrophic velocity  $\mathbf{v}$  at  $P$  and at the sea surface  $\mathbf{v}_0$  according to

$$\mathbf{k} \times \nabla_P \Psi = f\mathbf{v} - f\mathbf{v}_0. \quad (3.27.2)$$

Dynamic height anomaly is also commonly called the “geopotential anomaly”. The specific volume anomaly,  $\hat{\delta}$  in the vertical integral in Eqn. (3.27.1) could be replaced with specific volume  $\hat{v}$  without affecting the isobaric gradient of the resulting streamfunction. That is, this substitution would not affect Eqn. (3.27.2) because the additional term is a function only of pressure. Traditionally it was important to use specific volume anomaly in preference to specific volume as it was more accurate with computer code which worked with single-precision variables. Since computers now regularly employ double-precision, this issue has been overcome and consequently either  $\hat{\delta}$  or  $\hat{v}$  could be used in the integrand of Eqn. (3.27.1), so making it either the “dynamic height anomaly” or the “dynamic height”. As in the case of Eqn. (3.24.2), so also the dynamic height anomaly Eqn. (3.27.1) has not assumed that the gravitational acceleration is constant and so Eqn. (3.27.2) applies even when the gravitational acceleration is taken to vary in the vertical.

The dynamic height anomaly  $\Psi$  should be quoted in units of  $\text{m}^2 \text{s}^{-2}$ . These are the units in which the GSW Toolbox (appendix N) outputs dynamic height anomaly in the function `gsw_geo_strf_dyn_height(SA,CT,p,p_ref)`. When the last argument of this function, `p_ref`, is other than zero, the function returns the dynamic height anomaly with respect to a (deep) reference pressure `p_ref`, rather than with respect to  $P_0$  (i.e. zero dbar sea pressure) as in Eqn. (3.27.1). In this case the lateral gradient of the streamfunction represents the geostrophic velocity difference relative to the (deep)  $p_{\text{ref}}$  pressure surface, that is,

$$\mathbf{k} \times \nabla_P \Psi = f\mathbf{v} - f\mathbf{v}_{\text{ref}}. \quad (3.27.3)$$

Note that the integration in Eqn. (3.27.1) of specific volume anomaly with pressure must be done with pressure in Pa (not dbar) in order to have the resultant isobaric gradient,  $\nabla_P \Psi$ , in the usual units, being the product of the Coriolis parameter (units of  $\text{s}^{-1}$ ) and the velocity (units of  $\text{m s}^{-1}$ ).