Notes on the function, gsw_adiabatic_lapse_rate_from_CT(SA, CT, p), which evaluates the adiabatic lapse rate from Conservative Temperature

This function, **gsw_adiabatic_lapse_rate_from_CT**, finds the adiabatic lapse rate from given values of Absolute Salinity, Conservative Temperature and pressure. This function first finds the *in situ* temperature t from the GSW function **gsw_t_from_CT**(SA,CT,p), and then the adiabatic lapse rate is calculated directly from $\Gamma = -\eta_P/\eta_T = -g_{TP}/g_{TT}$ which comes from Eqn. (2.22.1) of the TEOS-10 Manual (IOC et al. (2010),

$$\Gamma = \Gamma\left(S_{\mathrm{A}},t,p\right) = \left.\frac{\partial t}{\partial P}\right|_{S_{\mathrm{A}},\eta} = \left.\frac{\partial t}{\partial P}\right|_{S_{\mathrm{A}},\Theta} = -\frac{g_{TP}}{g_{TT}} = \left.\frac{\partial^2 h}{\partial\eta\partial P}\right|_{S_{\mathrm{A}}} = \left.\frac{\partial v}{\partial\eta}\right|_{S_{\mathrm{A}},p} = \frac{\left(T_0+t\right)\alpha^t}{\rho c_p}$$

$$= \frac{\left(T_0+\theta\right)}{c_p^0}\left.\frac{\partial v}{\partial\Theta}\right|_{S_{\mathrm{A}},p} = \frac{\left(T_0+\theta\right)}{c_p^0}\left.\frac{\partial^2 h}{\partial\Theta\partial P}\right|_{S_{\mathrm{A}}} = \frac{\left(T_0+\theta\right)\alpha^\Theta}{\rho c_p^0} = \frac{\left(T_0+\theta\right)\alpha^\theta}{\rho c_p\left(S_{\mathrm{A}},\theta,0\right)}. \qquad (2.22.1)$$

The partial derivatives of the Gibbs function g_{TP} and g_{TT} are calculated using the library function **gsw_gibbs** which contains the Gibbs function $g\left(S_{\mathrm{A}},t,p\right)$ and its derivatives (the Gibbs function of seawater being the sum of those of IAPWS-08 and IAPWS-09).

<u>References</u>

IAPWS, 2008: Release on the IAPWS Formulation 2008 for the Thermodynamic Properties of Seawater. The International Association for the Properties of Water and Steam. Berlin, Germany, September 2008, available from www.iapws.org. This Release is referred to in the text as **IAPWS-08**.

IAPWS, 2009: Supplementary Release on a Computationally Efficient Thermodynamic Formulation for Liquid Water for Oceanographic Use. The International Association for the Properties of Water and Steam. Doorwerth, The Netherlands, September 2009, available from http://www.iapws.org. This Release is referred to in the text as **IAPWS-09**.

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org

Below is copied section 2.22 of the TEOS-10 Manual.

2.22 The adiabatic lapse rate

The adiabatic lapse rate Γ is the change of *in situ* temperature with pressure at constant entropy and Absolute Salinity, so that (McDougall and Feistel (2003))

$$\Gamma = \Gamma(S_{\mathrm{A}},t,p) = \left.\frac{\partial t}{\partial P}\right|_{S_{\mathrm{A}},\eta} = \left.\frac{\partial t}{\partial P}\right|_{S_{\mathrm{A}},\Theta} = -\frac{g_{TP}}{g_{TT}} = \left.\frac{\partial^2 h}{\partial\eta\partial P}\right|_{S_{\mathrm{A}}} = \left.\frac{\partial v}{\partial\eta}\right|_{S_{\mathrm{A}},p} = \frac{(T_0+t)\,\alpha^t}{\rho\, c_p}$$

$$= \frac{(T_0+\theta)}{c_p^0}\left.\frac{\partial v}{\partial\Theta}\right|_{S_{\mathrm{A}},p} = \frac{(T_0+\theta)}{c_p^0}\left.\frac{\partial^2 h}{\partial\Theta\partial P}\right|_{S_{\mathrm{A}}} = \frac{(T_0+\theta)\,\alpha^\Theta}{\rho\, c_p^0} = \frac{(T_0+\theta)\,\alpha^\theta}{\rho\, c_p(S_{\mathrm{A}},\theta,0)}\,. \qquad (2.22.1)$$

The adiabatic (and isohaline) lapse rate is commonly (and incorrectly) explained as being proportional to the work done on a fluid parcel as its volume changes in response to an increase in pressure. According to this explanation the adiabatic lapse rate would increase with both pressure and the fluid's compressibility, but this is not the case. Rather, the adiabatic lapse rate is proportional to the thermal expansion coefficient and is independent of the fluid's compressibility. Indeed, the adiabatic lapse rate changes sign at the temperature of maximum density whereas the compressibility and the work done by compression is always positive. McDougall and Feistel (2003) show that the adiabatic lapse rate is independent of the increase in the internal energy that a parcel experiences when it is compressed. Rather, the adiabatic lapse rate represents that change in temperature that is required to keep the entropy (and also θ and Θ) of a seawater parcel constant when its pressure is changed in an adiabatic and isohaline manner. The reference pressure of the potential temperature θ that appears in the last four expressions in Eqn. (2.22.1) is $p_{\mathrm{r}} = 0$ dbar.

The adiabatic lapse rate Γ in the GSW computer software library is evaluated via the functions **gsw_adiabatic_lapse_rate_from_t** and **gsw_adiabatic_lapse_rate_from_CT** (depending on whether the input temperature is *in situ* temperature or Conservative Temperature). In both cases the expression used is $-g_{TP}/g_{TT} = -\eta_P/\eta_T$ (see the top line of Eqn. (2.22.1)) calculated directly from the Gibbs function of seawater $g(S_{\mathrm{A}},t,p)$ (IAPWS-08 and IAPWS-09). This is consistent with the exact use of $\eta = \eta(S_{\mathrm{A}},t,p)$ throughout the GSW Toolbox to convert between *in situ* temperature and potential temperature. An alternative option for calculating Γ would be to use the 48-term expression for specific volume in the expressions in the second line of Eqn. (2.22.1). This option is not adopted as it would mean that the small errors in the thermal expansion coefficient α^Θ would cause an rms error in the adiabatic lapse rate Γ of $4.7x10^{-12}\ \mathrm{K\,Pa}^{-1}$. This error, while small, would then conflict with the exact relationships that have been chosen to relate *in situ* temperature, potential temperature, Conservative Temperature, entropy and the adiabatic lapse rate.

The adiabatic lapse rate Γ output of both the SIA and the GSW computer software libraries is in units of $\mathrm{K\,Pa}^{-1}$.

Reference:

McDougall, T. J. and R. Feistel, 2003: What causes the adiabatic lapse rate? *Deep-Sea Research,* **50**, 1523-1535.