Notes on the function gsw_Nsquared(SA, CT, p, {lat}) which evaluates N^2, the square of the buoyancy frequency

This function, **gsw_Nsquared**(SA,CT,p,{lat}), evaluates N^2, the square of the buoyancy frequency, on a vertical cast using the the 75-term polynomial function expression for specific volume **gsw_specvol**(SA,CT,p). The last argument, latitude, is optional. This 75-term polynomial expression for specific volume is discussed in Roquert *et al*. (2015) and in appendix A.30 and appendix K of the TEOS-10 Manual (IOC *et al*. (2010)). For dynamical oceanography we may take the 75-term polynomial expression for specific volume as essentially reflecting the full accuracy of TEOS-10.

This function, **gsw_Nsquared**(SA,CT,p,{lat}), evaluates N^2 from Eqn. (3.10.2) of the TEOS-10 Manual, that is, by using the equation

$$N^2 = g^2 \rho \frac{\beta^\Theta \Delta S_A - \alpha^\Theta \Delta\Theta}{\Delta P}$$

where ΔS_A and $\Delta\Theta$ are the differences between the Absolute Salinities and Conservative Temperatures of vertically adjacent seawater parcels separated in pressure by ΔP, measured in Pa. The density ρ and the saline contraction and thermal expansion coefficients β^Θ and α^Θ are evaluated at the average values of S_A, Θ and P of the two seawater parcels using the GSW Toolbox function **gsw_rho_alpha_beta**(SA,CT,p). The gravitational acceleration g is found from the GSW Toolbox function **gsw_grav**(lat, p) which is a function of latitude and of pressure in the ocean.

References

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org

Roquet, F., G. Madec, T. J. McDougall and P. M. Barker, 2015: Accurate polynomial expressions for the density and specific volume of seawater using the TEOS-10 standard. Ocean Modelling, 90, pp. 29-43. http://dx.doi.org/10.1016/j.ocemod.2015.04.002

Here follows section 3.10 of the TEOS-10 Manual (IOC *et al*. (2010)).

3.10 Buoyancy frequency

The square of the buoyancy frequency (sometimes called the Brunt-Väisälä frequency) N^2 is given in terms of the vertical gradients of density and pressure, or in terms of the vertical gradients of Conservative Temperature and Absolute Salinity (or in terms of the vertical gradients of potential temperature and Absolute Salinity) by (the g on the left-hand side is the gravitational acceleration, and x, y and z are the spatial Cartesian coordinates)

$$\begin{aligned} g^{-1}N^2 &= -\rho^{-1}\rho_z + \kappa P_z \;=\; -\rho^{-1}\left(\rho_z - P_z / c^2\right) \\ &= \alpha^{\theta}\theta_z - \beta^{\theta}\,\partial S_{\mathrm{A}}/\partial z\big|_{x,y} \\ &= \alpha^{\Theta}\Theta_z - \beta^{\Theta}\,\partial S_{\mathrm{A}}/\partial z\big|_{x,y} . \end{aligned} \tag{3.10.1}$$

For two seawater parcels separated by a small distance Δz in the vertical, an equally accurate method of calculating the buoyancy frequency is to bring both seawater parcels adiabatically and without exchange of matter to the average pressure and to calculate the difference in density of the two parcels after this change in pressure. In this way the potential density of the two seawater parcels are being compared at the same pressure. This common procedure calculates the buoyancy frequency N according to

$$N^2 = g\left(\alpha^{\Theta}\Theta_z - \beta^{\Theta}S_{\mathrm{A}_z}\right) \approx -\frac{g}{\rho}\frac{\Delta\rho^{\Theta}}{\Delta z}, \quad \text{or} \quad N^2 = g^2\rho\left(\beta^{\Theta}S_{\mathrm{A}_P} - \alpha^{\Theta}\Theta_P\right) \approx \frac{g^2\,\Delta\rho^{\Theta}}{\Delta P}, \tag{3.10.2}$$

where $\Delta\rho^{\Theta}$ is the difference between the potential densities of the two seawater parcels with the reference pressure being the average of the two original pressures of the seawater parcels. Eqn. (3.10.2b) has made use of the hydrostatic relation $P_z = -g\rho$.