Notes on the function gsw_pot_rho_t_exact(SA,t,p,pr)

This function, **gsw_pot_rho_t_exact**(SA,t,p,pr), evaluates the potential density with respect to reference pressure p_r for given input values of Absolute Salinity S_A, *in situ* temperature t, and pressure p. This function uses the full TEOS-10 Gibbs function $g(S_\mathrm{A},t,p)$ of IOC *et al*. (2010), being the sum of the IAPWS-09 and IAPWS-08 Gibbs functions. The potential density is evaluated directly from the Gibbs function using Eqn. (3.4.2) of the TEOS-10 Manual (IOC *et al*., 2010), repeated here,

$$\rho^\theta(S_\mathrm{A},t,p,p_\mathrm{r}) = \rho\left(S_\mathrm{A},\theta[S_\mathrm{A},t,p,p_\mathrm{r}],p_\mathrm{r}\right) = g_P^{-1}\left(S_\mathrm{A},\theta[S_\mathrm{A},t,p,p_\mathrm{r}],p_\mathrm{r}\right). \qquad (3.4.2)$$

This function **gsw_pot_rho_t_exact**(SA,t,p,pr), amounts to the following two calls to other GSW functions

```
pt = gsw_pt_from_t(SA,t,p,pr);
pot_rho_t_exact = gsw_rho_t_exact(SA,pt,pr);
```

References

IAPWS, 2008: Release on the IAPWS Formulation 2008 for the Thermodynamic Properties of Seawater. The International Association for the Properties of Water and Steam. Berlin, Germany, September 2008, available from www.iapws.org. This Release is referred to in the text as **IAPWS-08**.

IAPWS, 2009: Supplementary Release on a Computationally Efficient Thermodynamic Formulation for Liquid Water for Oceanographic Use. The International Association for the Properties of Water and Steam. Doorwerth, The Netherlands, September 2009, available from http://www.iapws.org. This Release is referred to in the text as **IAPWS-09**.

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org

Here follows section 3.4 of the TEOS-10 Manual (IOC *et al*., 2010).

3.4 Potential density

Potential density ρ^θ is the density that a fluid parcel would have if its pressure were changed to a fixed reference pressure p_r in an isentropic and isohaline manner. Potential density referred to reference pressure p_r can be written as the pressure integral of the isentropic compressibility κ as

$$\rho^\theta(S_\mathrm{A},t,p,p_\mathrm{r}) = \rho(S_\mathrm{A},t,p) + \int_P^{P_\mathrm{r}} \rho\left(S_\mathrm{A},\theta[S_\mathrm{A},t,p,p'],p'\right)\,\kappa\left(S_\mathrm{A},\theta[S_\mathrm{A},t,p,p'],p'\right)dP'. \qquad (3.4.1)$$

The simpler expression for potential density in terms of the Gibbs function is

$$\rho^\theta(S_\mathrm{A},t,p,p_\mathrm{r}) = \rho\left(S_\mathrm{A},\theta[S_\mathrm{A},t,p,p_\mathrm{r}],p_\mathrm{r}\right) = g_P^{-1}\left(S_\mathrm{A},\theta[S_\mathrm{A},t,p,p_\mathrm{r}],p_\mathrm{r}\right). \qquad (3.4.2)$$

Using the functional forms of Eqn. (2.8.2) and (2.8.3) for *in situ* density, that is, either $\rho = \tilde{\rho}(S_\mathrm{A},\theta,p)$ or $\rho = \hat{\rho}(S_\mathrm{A},\Theta,p)$, potential density with respect to reference pressure p_r (e. g. 1000 dbar) can be easily evaluated as

$$\rho^{\theta}\left(S_{\mathrm{A}},t,p,p_{\mathrm{r}}\right) = \rho^{\Theta}\left(S_{\mathrm{A}},t,p,p_{\mathrm{r}}\right) = \bar{\rho}\left(S_{\mathrm{A}},\eta,\ p_{\mathrm{r}}\right) = \tilde{\rho}\left(S_{\mathrm{A}},\theta,\ p_{\mathrm{r}}\right) = \hat{\rho}\left(S_{\mathrm{A}},\Theta,\ p_{\mathrm{r}}\right), \qquad (3.4.3)$$

where we note that the potential temperature θ in the penultimate expression is the potential temperature with respect to 0 dbar. Once the reference pressure is fixed, potential density is a function only of Absolute Salinity and Conservative Temperature (or equivalently, of Absolute Salinity and potential temperature). Note that it is equally correct to label potential density as ρ^{θ} or ρ^{Θ} (or indeed as ρ^{η}) because η , θ and Θ are constant during the isentropic and isohaline pressure change from p to p_{r}; that is, these variables posses the "potential" property of appendix A.9.

Following the discussion after Eqn. (2.8.2) above, potential density may also be expressed in terms of the pressure derivative of the expressions $h = \tilde{h}\left(S_{\mathrm{A}},\theta,p\right)$ and $h = \hat{h}\left(S_{\mathrm{A}},\Theta,p\right)$ for enthalpy as (see also appendix P)

$$\rho^{\theta}\left(S_{\mathrm{A}},t,p,p_{\mathrm{r}}\right) = \rho^{\Theta}\left(S_{\mathrm{A}},t,p,p_{\mathrm{r}}\right) = \left[\tilde{h}_{P}\left(S_{\mathrm{A}},\theta,\ p{=}\,p_{\mathrm{r}}\right)\right]^{-1} = \left[\hat{h}_{P}\left(S_{\mathrm{A}},\Theta,\ p{=}\,p_{\mathrm{r}}\right)\right]^{-1}. \qquad (3.4.4)$$