Notes on the function
gsw_stabilise_SA_const_t (SA,CT,p)

Stabilisation of hydrographic profiles

1. Introduction

The vertical stability of a water column is described by the square of the buoyancy frequency (N^2). In this paper we follow the N^2 definition similar to that used in Jackett and McDougall (1995), now defined in terms of the vertical gradients of Absolute Salinity and Conservative Temperature (these being the salinity and temperature variables of TEOS-10, IOC et al. (2010))

$$g^{-2}N^2 \;=\; -\frac{\alpha^{\Theta}}{v}\frac{\partial\Theta}{\partial P}\bigg|_{x,y} \;+\; \frac{\beta^{\Theta}}{v}\frac{\partial S_{\mathrm{A}}}{\partial P}\bigg|_{x,y}\,, \tag{1}$$

where the vertical derivatives are taken at constant latitude and longitude with respect to absolute pressure P. The gravitational acceleration is given the symbol g and v is the specific volume of seawater, being the reciprocal of in situ density ρ, and the thermal expansion and haline contraction coefficients defined with respect to Absolute Salinity S_{A} and Conservative Temperature Θ are given by

$$\alpha^{\Theta} \;=\; \frac{1}{v}\frac{\partial v}{\partial\Theta}\bigg|_{S_{\mathrm{A}},\,p} \qquad \text{and} \qquad \beta^{\Theta} \;=\; -\frac{1}{v}\frac{\partial v}{\partial S_{\mathrm{A}}}\bigg|_{\Theta,\,p}\,. \tag{2}$$

There are many ways to evaluate N^2 and McDougall and Barker (2014) list six of the most commonly used definitions and they show that they are all equivalent. In this paper we will also use the expression for N^2 written in terms of the vertical gradient of in situ temperature, namely

$$g^{-2}N^2 \;=\; -\frac{\alpha^{t}}{v}\left[\frac{\partial T}{\partial P}\bigg|_{x,y} - \Gamma\right] \;+\; \frac{\beta^{t}}{v}\frac{\partial S_{\mathrm{A}}}{\partial P}\bigg|_{x,y}\,, \tag{3}$$

where Γ is the adiabatic lapse rate, and the thermal expansion and haline contraction coefficients defined with respect to Absolute Salinity S_{A} and in situ temperature are given by

$$\alpha^t = \frac{1}{v}\frac{\partial v}{\partial T}\bigg|_{S_A,\,p} \qquad \text{and} \qquad \beta^t = -\frac{1}{v}\frac{\partial v}{\partial S_A}\bigg|_{T,\,p} . \tag{4}$$

Throughout this paper we follow the naming convention used in Jackett and McDougall (1995) so that we use the word "cast" to describe a vertical profile of either hydrographic data or data from a gridded product, and the word "bottle" to describe a data point at a particular pressure on such a cast. An instability is detected on a part of the cast where the square of the buoyancy frequency evaluated between a bottle pair is negative.

2. Stabilisation by adjusting only Absolute Salinity, keeping in situ temperature unchanged

A single cast of observed data contains values of Absolute Salinity, in situ temperature, and pressure p, for n bottles, that is $\left(S_{Aj}, t_j, p_j\right)$, $j = 1, 2, \ldots, n$, and the first step is to evaluate N^2 for each bottle pair. We want to adjust only the Absolute Salinity of the bottles while keeping the bottles' in situ temperatures constant. The values of N^2 down the cast can be compared to the lower limit that we specify, $N^2_{lower\_limit}$, as follows

$$b = \frac{\Delta P\, v}{\beta^t g^2}\left(N^2 - N^2_{lower\_limit}\right). \tag{5}$$

where $\Delta P (= \left(10^4\,\mathrm{Pa/dbar}\right)\Delta p)$ are the differences between the absolute pressures between adjacent bottles (deepest minus shallowest). The values of b for each bottle pair will be positive if the water column is more stable than our lower limit at this depth interval, and if the value of b is negative for a bottle pair, then the Absolute Salinities of one or other (or both) of the bottles of the pair will need to be altered in order to increase the value of N^2.

The motivation for the form (5) comes from Eqn. (3), realising that we want to minimize the perturbations of Absolute Salinity x_j (that is, we minimize $x_j x_j$) while obeying the inequality constraint

$$\mathrm{A}_{ij} x_j \le b_i , \tag{6}$$

with the matrix A_{ij} being a sparse double-striped matrix of values of +1 and -1 representing the differences of the perturbation Absolute Salinities for each bottle pair, and the values of

b_i for each bottle pair being evaluated as in Eqn. (5). Eqn. (3) suggests that if both the in situ temperatures and the values of the adiabatic lapse rate remain unchanged, then Eqns. (5) and (6) should give an accurate estimate of the required changes in Absolute Salinity for each bottle.

This algorithm is available as part of the Gibbs SeaWater Oceanographic Toolbox (from version 3.06) as the function *gsw_stabilise_SA_const_t*. In the standard form of this function we set $N^2_{lower\_limit}$ for each bottle pair to be $1x10^{-9}\,\mathrm{s}^{-2}$ which is approximately one fifth of the square of the earth's rotation rate. If desired, a vector of values of $N^2_{lower\_limit}$ can be supplied, one for each bottle pair, and in this case, the code will ensure that the resulting stabilised cast is at least as stable as this specified vertical profile. This method of stabilizing a water column is motivated by the idea that the measured in situ temperature is error free but that the Absolute Salinity contains salinity spiking errors or anomalies due to biological material passing through the conductivity sensor.