Notes on the function, gsw_entropy_from_CT(SA, CT), which evaluates specific entropy from Conservative Temperature

This function, **gsw_entropy_from_CT**, finds $\eta = \eta\left(S_{\mathrm{A}},\Theta\right)$, specific entropy as a function of Absolute Salinity and Conservative Temperature. This is done by first evaluating potential temperature θ (with reference pressure $p_{\mathrm{r}} = 0\ \mathrm{dbar}$) from the function **gsw_pt_from_CT** and then calling the temperature derivative of the Gibbs function as follows,

$$\eta \;=\; \tilde{\eta}\left(S_{\mathrm{A}},\theta\right) \;=\; -\,g_{T}\left(S_{\mathrm{A}},\theta,\, p=0\right). \tag{1}$$

Here follows appendix A.10 of the TEOS-10 Manual (IOC *et al*. (2010)).

A.10 Proof that $\theta = \theta\left(S_{\mathrm{A}},\eta\right)$ and $\Theta = \Theta\left(S_{\mathrm{A}},\theta\right)$

Consider changes occurring at the sea surface, (specifically at $p = 0$ dbar) where the temperature is the same as the potential temperature referenced to 0 dbar and the increment of pressure $\mathrm{d}p$ is zero. Regarding specific enthalpy h and chemical potential μ to be functions of entropy η (in place of temperature t), that is, considering the functional form of h and μ to be $h = \hat{h}\left(S_{\mathrm{A}},\eta,p\right)$ and $\mu = \hat{\mu}\left(S_{\mathrm{A}},\eta,p\right)$, it follows from the fundamental thermodynamic relation (Eqn. (A.7.1)) that

$$\hat{h}_{\eta}\left(S_{\mathrm{A}},\eta,0\right)\,\mathrm{d}\eta \;+\; \hat{h}_{S_{\mathrm{A}}}\left(S_{\mathrm{A}},\eta,0\right)\,\mathrm{d}S_{\mathrm{A}} \;=\; \left(T_0+\theta\right)\mathrm{d}\eta \;+\; \mu\left(S_{\mathrm{A}},\eta,0\right)\,\mathrm{d}S_{\mathrm{A}}\,, \tag{A.10.1}$$

which shows that specific entropy η is simply a function of Absolute Salinity S_{A} and potential temperature θ, that is $\eta = \eta\left(S_{\mathrm{A}},\theta\right)$, with no separate dependence on pressure. It follows that $\theta = \theta\left(S_{\mathrm{A}},\eta\right)$.

Similarly, from the definition of potential enthalpy and Conservative Temperature in Eqns. (3.2.1) and (3.3.1), at $p = 0$ dbar it can be seen that the fundamental thermodynamic relation (A.7.1) implies

$$c_p^0\,\mathrm{d}\Theta \;=\; \left(T_0+\theta\right)\mathrm{d}\eta \;+\; \tilde{\mu}\left(S_{\mathrm{A}},\theta,0\right)\,\mathrm{d}S_{\mathrm{A}}\,. \tag{A.10.2}$$

This shows that Conservative Temperature is also simply a function of Absolute Salinity and potential temperature, $\Theta = \Theta\left(S_{\mathrm{A}},\theta\right)$, with no separate dependence on pressure. It then follows that Θ may also be expressed as a function of only S_{A} and η. It follows that Θ has the "potential" property.