Notes on the function gsw_specvol_t_exact(SA,t,p)

This function, **gsw_specvol_t_exact**(SA,t,p), evaluates the specific volume v for given input values of Absolute Salinity S_A, *in situ* temperature t, and pressure p. This function uses the full TEOS-10 Gibbs function $g(S_A, t, p)$ of IOC *et al.* (2010), being the sum of the IAPWS-09 and IAPWS-08 Gibbs functions. The specific volume is evaluated directly from the Gibbs function using Eqn. (2.7.1) of the TEOS-10 Manual (IOC *et al.*, 2010), repeated here,

$$v = v(S_A, t, p) = g_P = \partial g / \partial P \big|_{S_A, T} . \tag{2.7.1}$$

References

IAPWS, 2008: Release on the IAPWS Formulation 2008 for the Thermodynamic Properties of Seawater. The International Association for the Properties of Water and Steam. Berlin, Germany, September 2008, available from www.iapws.org. This Release is referred to in the text as **IAPWS-08**.

IAPWS, 2009: Supplementary Release on a Computationally Efficient Thermodynamic Formulation for Liquid Water for Oceanographic Use. The International Association for the Properties of Water and Steam. Doorwerth, The Netherlands, September 2009, available from http://www.iapws.org. This Release is referred to in the text as **IAPWS-09**.

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org

Here follows sections 2.7 and 2.8 of the TEOS-10 Manual (IOC *et al.*, 2010).

2.7 Specific volume

The specific volume of seawater v is given by the pressure derivative of the Gibbs function at constant Absolute Salinity S_A and *in situ* temperature t, that is

$$v = v(S_A, t, p) = g_P = \partial g / \partial P \big|_{S_A, T} . \tag{2.7.1}$$

Notice that specific volume is a function of Absolute Salinity S_A rather than of Reference Salinity S_R or Practical Salinity S_P. The importance of this point is discussed in section 2.8. When derivatives are taken with respect to *in situ* temperature, or at constant *in situ* temperature, the symbol t is avoided as it can be confused with the same symbol for time. Rather, we use T in place of t in the expressions for these derivatives.

For many theoretical and modeling purposes in oceanography it is convenient to regard the independent temperature variable to be Conservative Temperature Θ rather than *in situ* temperature t. We note here that the specific volume is equal to the pressure derivative of specific enthalpy at fixed Absolute Salinity when any one of η, θ or Θ is also held constant, as follows (from appendix A.11)

$$\partial h / \partial P \big|_{S_A, \eta} = \partial h / \partial P \big|_{S_A, \Theta} = \partial h / \partial P \big|_{S_A, \theta} = v . \tag{2.7.2}$$

The use of P in these equations emphasizes that it must be in Pa not dbar. Specific volume v has units of $m^3 \ kg^{-1}$ in both the SIA and GSW software libraries.

2.8 Density

The density of seawater ρ is the reciprocal of the specific volume. It is given by the reciprocal of the pressure derivative of the Gibbs function at constant Absolute Salinity S_A and *in situ* temperature t, that is

$$\rho = \rho(S_A, t, p) = (g_P)^{-1} = \left(\partial g / \partial P \big|_{S_A, T} \right)^{-1}. \tag{2.8.1}$$

Notice that density is a function of Absolute Salinity S_A rather than of Reference Salinity S_R or Practical Salinity S_P. This is an extremely important point because Absolute Salinity S_A in units of g kg^{-1} is numerically greater than Practical Salinity by between 0.165 g kg^{-1} and 0.195 g kg^{-1} in the open ocean so that if Practical Salinity were inadvertently used as the salinity argument for the density algorithm, a significant density error of between 0.12 kg m^{-3} and 0.15 kg m^{-3} would result.

For many theoretical and modeling purposes in oceanography it is convenient to regard density to be a function of Conservative Temperature Θ rather than of *in situ* temperature t. That is, it is convenient to form the following two functional forms of density,

$$\rho = \hat{\rho}(S_A, \Theta, p), \tag{2.8.2}$$

where Θ is Conservative Temperature. We will adopt the convention (see Table L.2 in appendix L) that when enthalpy h, specific volume v or density ρ are taken to be functions of potential temperature they attract an over-tilde as in $\tilde{v}$ or $\tilde{\rho}$, and when they are taken to be functions of Conservative Temperature they attract a caret as in $\hat{v}$ and $\hat{\rho}$. With this convention, expressions involving partial derivatives such as (2.7.2) can be written more compactly as (from appendix A.11)

$$\widehat{h}_P = \tilde{h}_P = \hat{h}_P = v = \rho^{-1} \tag{2.8.3}$$

since the other variables are taken to be constant during the partial differentiation. Appendix P lists expressions for many thermodynamic variables in terms of the thermodynamic potentials

$$h = \widehat{h}(S_A, \eta, p), \ h = \tilde{h}(S_A, \theta, p) \text{ and } h = \hat{h}(S_A, \Theta, p). \tag{2.8.4}$$

Density ρ has units of kg m^{-3} in both the SIA and GSW software libraries.

Computationally efficient expressions for $\hat{\rho}(S_A, \Theta, p)$ and $\tilde{\rho}(S_A, \theta, p)$ involving 48 coefficients are available (McDougall *et al.* (2011b)) and are described in appendix A.30 and appendix K. These expressions can be integrated with respect to pressure to provide closed expressions for $\hat{h}(S_A, \Theta, p)$ and $\tilde{h}(S_A, \theta, p)$ (see Eqn. (A.30.6)).