Notes on the function gsw_CT_maxdensity(SA, p)

Notes written 15th May 2015

This function, **gsw_CT_maxdensity**(SA, p), calculates the Conservative Temperature at which the density of seawater is a maximum, at given values of Absolute Salinity and pressure. This function uses the 75-term polynomial function expression for specific volume **gsw_specvol**(SA,CT,p). This 75-term polynomial expression for specific volume is discussed in Roquet *et al*. (2015) and in appendix A.30 and appendix K of the TEOS-10 Manual (IOC *et al*. (2010)).For dynamical oceanography we may take the 75-term polynomial expression for specific volume as essentially reflecting the full accuracy of TEOS-10.

This function **gsw_CT_maxdensity**(SA, p) uses a modified Newton-Raphson iteration procedure (McDougall and Wotherspoon, 2014) to find the Conservative Temperature at which the thermal expansion coefficient with respect to Conservative Temperature, $\alpha^{\Theta}(S_A, \Theta, p)$, is zero.

References

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org

McDougall T. J. and S. J. Wotherspoon, 2014: A simple modification of Newton's method to achieve convergence of order $1+\sqrt{2}$. *Applied Mathematics Letters*, **29**, 20-25. http://dx.doi.org/10.1016/j.aml.2013.10.008

Here follows sections 3.42 of the TEOS-10 Manual (IOC *et al*. (2010)).

3.42 Temperature of maximum density

At about 4 °C and atmospheric pressure, pure water has a density maximum below which the thermal expansion coefficient and the adiabatic lapse rate change their signs (Röntgen (1892), McDougall and Feistel (2003)). At salinities higher than 23.8 g kg^{-1} the temperature of maximum density t_{MD} is below the freezing point t_f (Table 3.42.1). The seasonal and spatial interplay between density maximum and freezing point is highly important for the stratification stability and the seasonal deep convection for brackish estuaries with permanent vertical and lateral salinity gradients such as the Baltic Sea (Feistel *et al.* (2008b), Leppäranta and Myrberg (2009), Reissmann *et al.* (2009)).

The temperature of maximum density t_{MD} is computed from the condition of vanishing thermal expansion coefficient, that is, from the solution of the implicit equation for $t_{MD}(S_A, p)$,

$$g_{TP}(S_A, t_{MD}, p) = 0. \qquad (3.42.1)$$

The temperature of maximum density is available in the GSW Oceanographic Toolbox as function **gsw_t_maxdensity_exact**. Selected TEOS-10 values computed from Eqn. (3.42.1) are given in Table 3.42.1.

Table 3.42.1: Freezing temperature t_f and temperature of maximum density t_{MD} for air-free brackish seawater with absolute salinities S_A between 0 and 25 g kg^{-1}, computed at the surface pressure from TEOS-10. Values of t_{MD} in parentheses are less than the freezing temperature.

S_A g kg^{-1}	t_f °C	t_{MD} °C	S_A g kg^{-1}	t_f °C	t_{MD} °C	S_A g kg^{-1}	t_f °C	t_{MD} °C
0	+0.003	3.978	8.5	–0.456	2.128	17	–0.912	0.250
0.5	–0.026	3.868	9	–0.483	2.019	17.5	–0.939	0.139
1	–0.054	3.758	9.5	–0.509	1.909	18	–0.966	0.027
1.5	–0.081	3.649	10	–0.536	1.800	18.5	–0.994	–0.085
2	–0.108	3.541	10.5	–0.563	1.690	19	–1.021	–0.196
2.5	–0.135	3.432	11	–0.590	1.580	19.5	–1.048	–0.308
3	–0.162	3.324	11.5	–0.616	1.470	20	–1.075	–0.420
3.5	–0.189	3.215	12	–0.643	1.360	20.5	–1.102	–0.532
4	–0.216	3.107	12.5	–0.670	1.249	21	–1.130	–0.644
4.5	–0.243	2.999	13	–0.697	1.139	21.5	–1.157	–0.756
5	–0.269	2.890	13.5	–0.724	1.028	22	–1.184	–0.868
5.5	–0.296	2.782	14	–0.750	0.917	22.5	–1.212	–0.980
6	–0.323	2.673	14.5	–0.777	0.807	23	–1.239	–1.092
6.5	–0.349	2.564	15	–0.804	0.696	23.5	–1.267	–1.204
7	–0.376	2.456	15.5	–0.831	0.584	24	–1.294	(–1.316)
7.5	–0.403	2.347	16	–0.858	0.473	24.5	–1.322	(–1.428)
8	–0.429	2.238	16.5	–0.885	0.362	25	–1.349	(–1.540)