Notes on the GSW library function gsw_Hill_ratio_at_SP2(t)

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This library function, **gsw_Hill_ratio_at_SP2**(t), calculates the ratio by which the Hill *et al*. (1986) formula for Practical Salinity differs from that of the Practical Salinity Scale of 1978 (PSS-78, (Unesco (1981, 1983)) at a Practical Salinity of 2 and at the given input temperature t (°C , ITS-90).

Practical Salinity (SP) is calculated in terms of the conductivity ratio (R_t) defined as (see Eqn. (E.2.1) of IOC *et al*. (2010))

$$R_t = \frac{C(S_P, t_{68}, 0)}{C(35, t_{68}, 0)},$$

being the ratio of two conductivity values, one from a general seawater sample and the other being a sample of Standard SeaWater (having a Practical Salinity of exactly 35), both being measured at the same temperature. Practical Salinity S_P is then calculated from the PSS-78 expression, Eqn. (E.2.6) of IOC *et al*. (2010), repeated here

$$S_P = \sum_{i=0}^{5} a_i \left(R_t\right)^{i/2} + \frac{\left(t_{68}/°C - 15\right)}{\left[1 + k\left(t_{68}/°C - 15\right)\right]} \sum_{i=0}^{5} b_i \left(R_t\right)^{i/2}. \qquad \text{(E.2.6)}$$

This formula is only valid when the resulting Practical Salinity is between 2 and 42. When Practical Salinity is less than 2 the GSW Oceanographic Toolbox uses a modified form of the Hill *et al*. (1986) expression for Practical Salinity. The output of the present function **gsw_Hill_ratio_at_SP2**(t) provides the multiplicative modification to other functions in the GSW Toolbox.

The first step in this library function **gsw_Hill_ratio_at_SP2**(t) is to calculate the t_{68} temperature from the t_{90} input temperature using Eqn. (A.1.3) of IOC *et al*. (2010), repeated here

$$\left(t_{68}/°C\right) = 1.00024\left(t_{90}/°C\right). \qquad \text{(A.1.3)}$$

The PSS-78 expression Eqn. (E.2.6) is then solved for R_t at the known value of S_P of 2 using a modified Newton-Raphson iterative technique. With these values of R_t and t_{68}, the Hill *et al*. (1986) expression for Practical Salinity,

$$S_P = \sum_{i=0}^{5} a_i \left(R_t\right)^{i/2} + \frac{\left(t_{68}/°C - 15\right)}{\left[1 + k\left(t_{68}/°C - 15\right)\right]} \sum_{i=0}^{5} b_i \left(R_t\right)^{i/2}$$
$$- \frac{a_0}{\left(1 + 600R_t + 160000\left(R_t\right)^2\right)} - \frac{\left(t_{68}/°C - 15\right)}{\left[1 + k\left(t_{68}/°C - 15\right)\right]} \frac{b_0}{\left(1 + 10\left(R_t\right)^{i/2} + 100R_t + 1000\left(R_t\right)^{3/2}\right)},$$

is evaluated, and the ratio of 2 to this value of Practical Salinity is the output of this function, **gsw_Hill_ratio_at_SP2**. This ratio is used to modify the Hill *et al*. (1986) expression for Practical Salinity in several of the GSW functions. For example, this ratio is used in **gsw_SP_from_C** so that the Practical Salinity output of this function is a continuous function of conductivity when Practical Salinity transitions through the value 2. Note that the first line of the Hill *et al*. (1986) equation above is the PSS-78 expression and the second line contains the two correction terms of Hill *et al*. (1986) where a_0 and b_0 are the constants $a_0 = 0.008$ and $b_0 = 0.0005$ of PSS-78. .

References

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