Notes on the function gsw_isopycnal_vs_ntp_CT_ratio(SA,CT,p)

This function **gsw_isopycnal_vs_ntp_CT_ratio**(SA,CT,p) evaluates the "isopycnal temperature gradient ratio" defined by (from section 3.17 of the TEOS-10 Manual, IOC *et al.* (2010))

$$ G^\Theta \equiv \frac{\left[R_\rho - 1 \right]}{\left[R_\rho / r - 1 \right]} \; . \tag{3.17.4} $$

This is the ratio of the (parallel) gradient of Conservative Temperature in a potential density surface, $\nabla_\sigma \Theta$, to that in a neutral tangent plane, $\nabla_n \Theta$, since, from Eqn. (3.17.3) of the TEOS-10 Manual,

$$ \nabla_\sigma \Theta = \frac{r\left[R_\rho - 1 \right]}{\left[R_\rho - r \right]} \nabla_n \Theta = G^\Theta \nabla_n \Theta \; . \tag{3.17.3} $$

This function, **gsw_isopycnal_vs_ntp_CT_ratio**(SA,CT,p), uses the 75-term polynomial function expression for specific volume **gsw_specvol**(SA,CT,p). This 75-term polynomial expression for specific volume is discussed in Roquert *et al.* (2015) and in appendix A.30 and appendix K of the TEOS-10 Manual (IOC *et al.* (2010)).For dynamical oceanography we may take the 75-term polynomial expression for specific volume as essentially reflecting the full accuracy of TEOS-10.

References

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org

Roquet, F., G. Madec, T. J. McDougall and P. M. Barker, 2015: Accurate polynomial expressions for the density and specific volume of seawater using the TEOS-10 standard. Ocean Modelling, 90, pp. 29-43. http://dx.doi.org/10.1016/j.ocemod.2015.04.002

Here follows section 3.17 of the TEOS-10 Manual (IOC *et al.* (2010)).

3.17 Property gradients along potential density surfaces

The two-dimensional gradient of a scalar φ along a potential density surface $\nabla_\sigma \varphi$ is related to the corresponding gradient in the neutral tangent plane $\nabla_n \varphi$ by

$$ \nabla_\sigma \varphi = \nabla_n \varphi + \frac{\varphi_z}{\Theta_z} \frac{R_\rho \left[r - 1 \right]}{\left[R_\rho - r \right]} \nabla_n \Theta \tag{3.17.1} $$

(from McDougall (1987a)), where r is the ratio of the slope on the $S_A - \Theta$ diagram of an isoline of potential density with reference pressure p_r to the slope of a potential density surface with reference pressure p, and is defined by

$$ r = \frac{\alpha^\Theta \left(S_A, \Theta, p \right) / \beta^\Theta \left(S_A, \Theta, p \right)}{\alpha^\Theta \left(S_A, \Theta, p_r \right) / \beta^\Theta \left(S_A, \Theta, p_r \right)} \; . \tag{3.17.2} $$

Substituting $\varphi = \Theta$ into (3.17.1) gives the following relation between the (parallel) isopycnal and epineutral gradients of Θ

$$\nabla_\sigma \Theta = \frac{r\left[R_\rho - 1\right]}{\left[R_\rho - r\right]} \nabla_n \Theta = G^\Theta \nabla_n \Theta \qquad (3.17.3)$$

where the "isopycnal temperature gradient ratio"

$$G^\Theta \equiv \frac{\left[R_\rho - 1\right]}{\left[R_\rho / r - 1\right]} \qquad (3.17.4)$$

has been defined as a shorthand expression for future use. This ratio G^Θ is available in the GSW Toolbox from the algorithm **gsw_isopycnal_vs_ntp_CT_ratio**, while the ratio r of Eqn. (3.17.2) is available there as **gsw_isopycnal_slope_ratio**. Substituting $\varphi = S_A$ into Eqn. (3.17.1) gives the following relation between the (parallel) isopycnal and epineutral gradients of S_A

$$\nabla_\sigma S_A = \frac{\left[R_\rho - 1\right]}{\left[R_\rho - r\right]} \nabla_n S_A = \frac{G^\Theta}{r} \nabla_n S_A. \qquad (3.17.5)$$