Notes on the GSW function gsw_CT_from_pt

This function **gsw_CT_from_pt** calculates Conservative Temperature Θ for given input values of Absolute Salinity S_A (g kg^{-1}) and potential temperature θ ($°C$, ITS-90), this being the potential temperature referenced to the reference sea pressure $p_r = 0\,\text{dbar}$. Conservative Temperature Θ is proportional to potential enthalpy h^0 defined by Eqn. (3.2.2) of the TEOS-10 manual, repeated here,

$$h^0\left(S_A, t, p\right) = h\left(S_A, \theta, 0\right) = g\left(S_A, \theta, 0\right) - \left(T_0 + \theta\right) g_T\left(S_A, \theta, 0\right). \tag{3.2.2}$$

The present GSW function **gsw_CT_from_pt** could have evaluated potential enthalpy via two calls to the library function **gsw_gibbs**; one call to evaluate $g\left(S_A, \theta, 0\right)$ and the other to evaluate $g_T\left(S_A, \theta, 0\right)$.

Instead, we have collected like powers of the non-dimensional variables representing S_A and θ in $g\left(S_A, \theta, 0\right) - \left(T_0 + \theta\right) g_T\left(S_A, \theta, 0\right)$ so that **gsw_CT_from_pt** is a simple polynomial in terms of these non-dimensional variables. There are three reasons for doing this. First, since pressure is zero, we can avoid the several unnecessary multiplications involving various powers of zero. Second, the act of collecting together the like powers of the two remaining variables effectively halves the number of multiplications that are needed, so making the function more computationally efficient. Third, both $g\left(S_A, \theta, 0\right)$ and $g_T\left(S_A, \theta, 0\right)$ contain terms in the logarithm of S_A which exactly cancel from the combination $g\left(S_A, \theta, 0\right) - \left(T_0 + \theta\right) g_T\left(S_A, \theta, 0\right)$. Not having to calculate these logarithm terms saves computer time and also avoids numerical issues that would otherwise occur as S_A approaches zero (actually these terms are four terms in $S_A \ln\left(\sqrt{S_A}\right)$ which do individually go to zero as S_A vanishes).

Here follows sections 3.2 and 3.3 of the TEOS-10 manual (IOC *et al.* (2010)).

3.2 Potential enthalpy

Potential enthalpy h^0 is the enthalpy that a fluid parcel would have if its pressure were changed to a fixed reference pressure p_r in an isentropic and isohaline manner. Because heat fluxes into and out of the ocean occur mostly near the sea surface, the reference pressure for potential enthalpy is always taken to be $p_r = 0\,\text{dbar}$ (that is, at zero sea pressure). Potential enthalpy can be expressed as the pressure integral of specific volume as (from McDougall (2003) and see the discussion below Eqn. (2.8.2))

$$
\begin{aligned}
h^0\left(S_A, t, p\right) = h\left(S_A, \theta, 0\right) = \tilde{h}^0\left(S_A, \theta\right) &= h\left(S_A, t, p\right) - \int_{P_0}^{P} v\left(S_A, \theta\left[S_A, t, p, p'\right], p'\right) dP' \\
&= h\left(S_A, t, p\right) - \int_{P_0}^{P} \hat{v}\left(S_A, \eta, p'\right) dP' \\
&= h\left(S_A, t, p\right) - \int_{P_0}^{P} \tilde{v}\left(S_A, \theta, p'\right) dP' \\
&= h\left(S_A, t, p\right) - \int_{P_0}^{P} \hat{v}\left(S_A, \Theta, p'\right) dP',
\end{aligned}
\tag{3.2.1}
$$

and we emphasize that the pressure integrals here must be done with respect to pressure expressed in Pa rather than dbar. In terms of the Gibbs function, potential enthalpy h^0 is evaluated as

$$h^0\left(S_A, t, p\right) = h\left(S_A, \theta, 0\right) = g\left(S_A, \theta, 0\right) - \left(T_0 + \theta\right) g_T\left(S_A, \theta, 0\right). \tag{3.2.2}$$

3.3 Conservative Temperature

Conservative Temperature Θ is defined to be proportional to potential enthalpy according to

$$\Theta\left(S_\mathrm{A}, t, p\right) = \tilde{\Theta}\left(S_\mathrm{A}, \theta\right) = h^0\left(S_\mathrm{A}, t, p\right)\big/c_p^0 = \tilde{h}^0\left(S_\mathrm{A}, \theta\right)\big/c_p^0 \qquad (3.3.1)$$

where the value that is chosen for c_p^0 is motivated in terms of potential enthalpy evaluated at an Absolute Salinity of $S_\mathrm{SO} = 35\,u_\mathrm{PS} = 35.165\,04 \mathrm{\ g\,kg^{-1}}$ and at $\theta = 25\,°\mathrm{C}$ by

$$\frac{\left[h\left(S_\mathrm{SO},\,25°\mathrm{C},\,0\right) - h\left(S_\mathrm{SO},\,0°\mathrm{C},\,0\right)\right]}{\left(25 \mathrm{\ K}\right)} \approx 3991.867\,957\,119\,63 \mathrm{\ J\,kg^{-1}\,K^{-1}}, \qquad (3.3.2)$$

noting that $h\left(S_\mathrm{SO},\,0\,°\mathrm{C},\,0\mathrm{dbar}\right)$ is zero according to the way the Gibbs function is defined in (2.6.5). In fact we adopt the exact definition for c_p^0 to be the 15-digit value in (3.3.2), so that

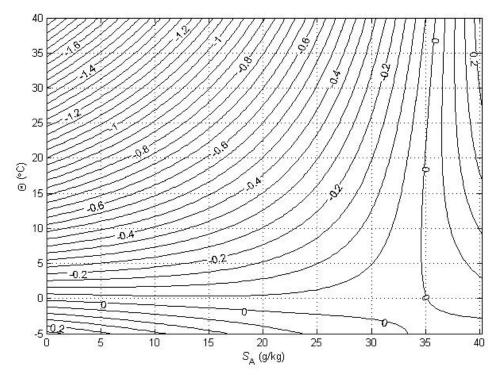$$c_p^0 \equiv 3991.867\,957\,119\,63 \mathrm{\ J\,kg^{-1}\,K^{-1}}. \qquad (3.3.3)$$

When IAPWS-95 is used for the pure water part of the Gibbs function, $\Theta\left(S_\mathrm{SO}, 0\,°\mathrm{C}, 0\right)$ and $\Theta\left(S_\mathrm{SO}, 25\,°\mathrm{C}, 0\right)$ differ from 0 °C and 25 °C respectively by the round-off amount of 5×10^{-12} °C. When IAPWS-09 (which is based on the paper of Feistel (2003), see appendix G) is used for the pure water part of the Gibbs function, $\Theta\left(S_\mathrm{SO}, 0\,°\mathrm{C}, 0\right)$ differs from 0 °C by -8.25×10^{-8} °C and $\Theta\left(S_\mathrm{SO}, 25\,°\mathrm{C}, 0\right)$ differs from 25 °C by 9.3×10^{-6} °C. Over the temperature range from 0°C to 40°C the difference between Conservative Temperature using IAPWS-95 and IAPWS-09 as the pure water part is no more than $\pm 1.5 \times 10^{-5}$ °C, a temperature difference that will be ignored.

The value of c_p^0 in (3.3.3) is very close to the average value of the specific heat capacity c_p at the sea surface of today's global ocean. This value of c_p^0 also causes the average value of $\theta - \Theta$ at the sea surface to be very close to zero. Since c_p^0 is simply a constant of proportionality between potential enthalpy and Conservative Temperature, it is totally arbitrary, and we see no reason why its value would need to change from (3.3.3) even when in future decades an improved Gibbs function of seawater is agreed upon.

Appendix A.18 outlines why Conservative Temperature gets its name; it is approximately two orders of magnitude more conservative compared with either potential temperature or entropy.

The SIA and GSW software libraries both include an algorithm for determining Conservative Temperature Θ from values of Absolute Salinity S_A and potential temperature θ referenced to $p = 0\,\mathrm{dbar}$. These libraries also have an algorithm for evaluating potential temperature (referenced to $0\,\mathrm{dbar}$) from S_A and Θ. This inverse algorithm, $\hat{\theta}\left(S_\mathrm{A}, \Theta\right)$, has an initial seed based on a rational function approximation and finds potential temperature to machine precision ($\sim 10^{-14}$ °C) in one and a half iterations of a modified Newton-Raphson technique (McDougall *et al.* (2011b)).

Also, note Figure A.17.1 below (from IOC *et al.* (2010)) showing the difference between potential temperature and Conservative Temperature.

Figure A.17.1. Contours (in °C) of the difference between potential temperature and Conservative Temperature $\theta - \Theta$. This plot illustrates the non-conservative production of potential temperature θ in the ocean.