3.17 Property gradients along potential density surfaces

The two-dimensional gradient of a scalar φ along a potential density surface $\nabla_\sigma \varphi$ is related to the corresponding gradient in the neutral tangent plane $\nabla_n \varphi$ by

$$\nabla_\sigma \varphi = \nabla_n \varphi + \frac{\varphi_z}{\Theta_z} \frac{R_\rho [r-1]}{[R_\rho - r]} \nabla_n \Theta \tag{3.17.1}$$

(from McDougall (1987a)), where r is the ratio of the slope on the $S_\mathrm{A} - \Theta$ diagram of an isoline of potential density with reference pressure p_r to the slope of a potential density surface with reference pressure p, and is defined by

$$r = \frac{\alpha^\Theta (S_\mathrm{A}, \Theta, p) / \beta^\Theta (S_\mathrm{A}, \Theta, p)}{\alpha^\Theta (S_\mathrm{A}, \Theta, p_\mathrm{r}) / \beta^\Theta (S_\mathrm{A}, \Theta, p_\mathrm{r})} \, . \tag{3.17.2}$$

Substituting $\varphi = \Theta$ into (3.17.1) gives the following relation between the (parallel) isopycnal and epineutral gradients of Θ

$$\nabla_\sigma \Theta = \frac{r [R_\rho - 1]}{[R_\rho - r]} \nabla_n \Theta = G^\Theta \nabla_n \Theta \tag{3.17.3}$$

where the "isopycnal temperature gradient ratio"

$$G^\Theta \equiv \frac{[R_\rho - 1]}{[R_\rho / r - 1]} \tag{3.17.4}$$

has been defined as a shorthand expression for future use. This ratio G^Θ is available in the GSW software library from the algorithm **gsw_isopycnal_vs_ntp_CT_ratio_CT25**, while the ratio r of Eqn. (3.17.2) is available there as **gsw_isopycnal_slope_ratio_CT25**. Substituting $\varphi = S_\mathrm{A}$ into Eqn. (3.17.1) gives the following relation between the (parallel) isopycnal and epineutral gradients of S_A

$$\nabla_\sigma S_\mathrm{A} = \frac{[R_\rho - 1]}{[R_\rho - r]} \nabla_n S_\mathrm{A} = \frac{G^\Theta}{r} \nabla_n S_\mathrm{A} . \tag{3.17.5}$$