Notes on the function gsw_geo_strf_dyn_height_pc

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In both z-coordinate and density-coordinate forward ocean models, the variables are interpreted as being piecewise constant in the vertical, and this function, gsw_geo_strf_dyn_height_pc, calculates the dynamic height anomaly taking this piecewise-constant nature of Absolute Salinity and Conservative Temperature into account. The code uses the computationally efficient 25-term rational function expression of McDougall *et al.* (2010) for the specific volume of seawater $\hat{v}(S_A, \Theta, p)$ in terms of Absolute Salinity S_A, Conservative Temperature Θ and pressure p.

In a so-called "z-coordinate" model the "thicknesses" of the layers are independent of latitude, longitude and time. Often the vertical coordinate is interpreted as pressure in these "z-coordinate" models, and in this case the "thickness" of the ith layer, $p^{i+1} - p^i$ is spatially and temporally invariant. In density-coordinate models (often called "isopycnal models" or "layered models") and also in hybrid-coordinate models, the "thickness" of the ith layer, $p^{i+1} - p^i$, varies with latitude, longitude and time. In these layered models the "thicknesses" of the least dense layers are often zero at many locations in the ocean, because these density surfaces have outcropped.

The pressure input to the function gsw_geo_strf_dyn_height_pc is the matrix delta_p = $p^{i+1} - p^i$ and the code can handle zero values of this layer "thickness" in several layers. The function calculates the pressures p_mid at the mid-pressure of each layer by summing the delta_p of the shallower layers and adding half of delta_p = $p^{i+1} - p^i$ at the ith layer. Taking the layer index i to increase from 1 in the upper-most layer, the Absolute Salinity and Conservative Temperature are constant at S_A^i and Θ^i in each layer, and the shallower and deeper pressures bounding the ith layer are p^i and p^{i+1} respectively.

The code gsw_geo_strf_dyn_height_pc uses the computationally efficient 25-term rational function expression of McDougall *et al.* (2010) for the specific volume of seawater $\hat{v}(S_A, \Theta, p)$ in terms of Absolute Salinity S_A, Conservative Temperature Θ and pressure p. The specific volume anomaly is defined with respect to $S_A = S_{SO} \equiv 35.165\,04 \text{ g kg}^{-1}$ and $\Theta = 0°\text{C}$ as

$$\hat{\delta}(S_A, \Theta, p) = \hat{v}(S_A, \Theta, p) - \hat{v}(S_{SO}, 0°\text{C}, p), \qquad (1)$$

and the thermodynamic identity

$$h_P\big|_{S_A,\Theta} = \hat{h}_P = \hat{v}, \qquad (2)$$

is used to write the dynamic height anomaly Ψ at the midpoint pressure $\tfrac{1}{2}\left[p^{n+1} + p^n\right]$ of layer n as

$$\begin{aligned}
\Psi &= -\int_{P_0}^{P} \hat{\delta}\left(S_A[p'], \Theta[p'], p'\right) dP' \\
&= -\int_{P_0}^{P} \hat{v}\left(S_A[p'], \Theta[p'], p'\right) dP' + \int_{P_0}^{P} \hat{v}\left(S_{SO}, \Theta = 0°\text{C}, p'\right) dP' \\
&= -\sum_{i=1}^{n-1}\left[\hat{h}\left(S_A^i, \Theta^i, p^{i+1}\right) - \hat{h}\left(S_A^i, \Theta^i, p^i\right)\right] \\
&\qquad - \hat{h}\left(S_A^n, \Theta^n, \tfrac{1}{2}\left[p^{n+1} + p^n\right]\right) + \hat{h}\left(S_A^n, \Theta^n, p^n\right) \\
&\qquad\quad + \hat{h}\left(S_{SO}, \Theta = 0°\text{C}, \tfrac{1}{2}\left[p^{n+1} + p^n\right]\right).
\end{aligned} \qquad (3)$$

Note the lower limit of the pressure integral of $\hat{v}(S_{SO}, 0°\text{C}, p')$ is $\hat{h}(S_{SO}, \Theta = 0°\text{C}, 0\,\text{dbar})$ which is zero (being c_p^0 times $\Theta = 0°\text{C}$).

The enthalpy differences in Eqn. (3) for pairs of seawater parcels having the same Absolute Salinity and Conservative Temperature but different values of pressure are obtained from the function gsw_enthalpy_diff_CT25 which has been designed to be computationally efficient. The last term in Eqn. (3) is found from the library function gsw_enthalpy_SSO_0_CT25 which is a function of only pressure; this function is a streamlined version gsw_enthalpy_CT25, simplified because the Conservative Temperature argument is zero.

References

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org See sections 3.27, 3.32 and appendix A.30.

McDougall T. J., D. R. Jackett, P. M. Barker, C. Roberts-Thomson, R. Feistel and R. W. Hallberg, 2010: A computationally efficient 25-term expression for the density of seawater in terms of Conservative Temperature, and related properties of seawater. To be submitted to *Ocean Science Discussions*.

Here follows section 3.27 from the TEOS-10 Manual (IOC *et al*. (2010)).

3.27 Dynamic height anomaly

The dynamic height anomaly Ψ, given by the vertical integral

$$\Psi = - \int_{P_0}^{P} \delta\left(S_{\mathrm{A}}[p'], t[p'], p'\right) dP', \qquad (3.27.1)$$

is the geostrophic streamfunction for the flow at pressure P with respect to the flow at the sea surface and δ is the specific volume anomaly. Thus the two-dimensional gradient of Ψ in the P pressure surface is simply related to the difference between the horizontal geostrophic velocity $\mathbf{v}$ at P and at the sea surface $\mathbf{v}_0$ according to

$$\mathbf{k} \times \nabla_P \Psi = f\mathbf{v} - f\mathbf{v}_0 . \qquad (3.27.2)$$

The definition Eqn. (3.27.1) of dynamic height anomaly applies to all choices of the reference values $\tilde{S}_{\mathrm{A}}$ and $\hat{t}$, $\tilde{\theta}$ or $\hat{\Theta}$ in the definition Eqns. (3.7.1 – 3.7.4) of the specific volume anomaly δ. Also, δ in Eqn. (3.27.1) can be replaced with specific volume v without affecting the isobaric gradient of the resulting streamfunction. That is, this substitution does not affect Eqn. (3.27.2) because the additional term is a function only of pressure. Traditionally it was important to use specific volume anomaly in preference to specific volume as it was more accurate with computer code which worked with single-precision variables. Since computers now regularly employ double-precision, this issue has been overcome and consequently either δ or v can be used in the integrand of Eqn. (3.27.1), so making it either the "dynamic height anomaly" or the "dynamic height". As in the case of Eqn. (3.24.2), so also the dynamic height anomaly Eqn. (3.27.1) has not assumed that the gravitational acceleration is constant and so Eqn. (3.27.2) applies even when the gravitational acceleration is taken to vary in the vertical.

The dynamic height anomaly Ψ should be quoted in units of $\mathrm{m^2\,s^{-2}}$. These are the units in which the GSW library (appendix N) outputs dynamic height anomaly in the function gsw_geo_strf_dyn_height. Note that the integration in Eqn. (3.27.1) of specific volume anomaly with pressure in dbar would yield dynamic height anomaly in units of $\mathrm{m^3\,kg^{-1}dbar}$, and the use of these units in Eqn. (3.27.2) would not give the resultant horizontal gradient in the usual units, being the product of the Coriolis parameter (units of $\mathrm{s^{-1}}$) and the velocity (units of $\mathrm{m\,s^{-1}}$). This is the reason why the pressure integration is done with pressure in Pa and dynamic height anomaly is output in $\mathrm{m^2\,s^{-2}}$.