Notes on the function, gsw_entropy_from_pt(SA, pt), which evaluates specific entropy from potential temperature with reference pressure of 0 dbar

This function, **gsw_entropy_from_pt**, finds $\eta = \eta(S_{\rm A},\theta)$, specific entropy as a function of Absolute Salinity and potential temperature (whose reference pressure $p_{\rm r} = 0$ dbar). This is evaluated directly from the Gibbs function as

$$\eta \;=\; \tilde{\eta}(S_{\rm A},\theta) \;=\; -\,g_T\,(S_{\rm A},\theta,\; p{=}0)\,. \tag{1}$$

Here follows appendix A.10 of the TEOS-10 Manual (IOC *et al*. (2010)).

A.10 Proof that $\theta = \theta(S_{\rm A},\eta)$ and $\Theta = \Theta(S_{\rm A},\theta)$

Consider changes occurring at the sea surface, (specifically at $p = 0$ dbar) where the temperature is the same as the potential temperature referenced to 0 dbar and the increment of pressure dp is zero. Regarding specific enthalpy h and chemical potential μ to be functions of entropy η (in place of temperature t), that is, considering the functional form of h and μ to be $h = \hat{h}(S_{\rm A},\eta,p)$ and $\mu = \hat{\mu}(S_{\rm A},\eta,p)$, it follows from the fundamental thermodynamic relation (Eqn. (A.7.1)) that

$$\hat{h}_{\eta}(S_{\rm A},\eta,0)\,{\rm d}\eta \;+\; \hat{h}_{S_{\rm A}}(S_{\rm A},\eta,0)\;{\rm d}S_{\rm A} \;=\; (T_0+\theta){\rm d}\eta \;+\; \mu(S_{\rm A},\eta,0)\,{\rm d}S_{\rm A}\,, \tag{A.10.1}$$

which shows that specific entropy η is simply a function of Absolute Salinity $S_{\rm A}$ and potential temperature θ, that is $\eta = \eta(S_{\rm A},\theta)$, with no separate dependence on pressure. It follows that $\theta = \theta(S_{\rm A},\eta)$.

Similarly, from the definition of potential enthalpy and Conservative Temperature in Eqns. (3.2.1) and (3.3.1), at $p = 0$ dbar it can be seen that the fundamental thermodynamic relation (A.7.1) implies

$$c_p^0\,{\rm d}\Theta \;=\; (T_0+\theta){\rm d}\eta \;+\; \tilde{\mu}(S_{\rm A},\theta,0)\,{\rm d}S_{\rm A}\,. \tag{A.10.2}$$

This shows that Conservative Temperature is also simply a function of Absolute Salinity and potential temperature, $\Theta = \Theta(S_{\rm A},\theta)$, with no separate dependence on pressure. It then follows that Θ may also be expressed as a function of only $S_{\rm A}$ and η. It follows that Θ has the "potential" property.