3.4 Potential density

Potential density ρ^{θ} is the density that a fluid parcel would have if its pressure were changed to a fixed reference pressure p_{r} in an isentropic and isohaline manner. Potential density referred to reference pressure p_{r} can be written as the pressure integral of the isentropic compressibility as

$$\rho^{\theta}\left(S_{\mathrm{A}},t,p,p_{\mathrm{r}}\right) = \rho\left(S_{\mathrm{A}},t,p\right) + \int_{P}^{p_{\mathrm{r}}} \rho\left(S_{\mathrm{A}},\theta\left[S_{\mathrm{A}},t,p,p'\right],p'\right)\,\kappa\left(S_{\mathrm{A}},\theta\left[S_{\mathrm{A}},t,p,p'\right],p'\right)dP'. \qquad (3.4.1)$$

The simpler expression for potential density in terms of the Gibbs function is

$$\rho^{\theta}\left(S_{\mathrm{A}},t,p,p_{\mathrm{r}}\right) = \rho\left(S_{\mathrm{A}},\theta\left[S_{\mathrm{A}},t,p,p_{\mathrm{r}}\right],p_{\mathrm{r}}\right) = g_P^{-1}\left(S_{\mathrm{A}},\theta\left[S_{\mathrm{A}},t,p,p_{\mathrm{r}}\right],p_{\mathrm{r}}\right). \qquad (3.4.2)$$

Using either of the functional forms (2.8.2) for *in situ* density, that is, either $\rho = \tilde{\rho}\left(S_{\mathrm{A}},\theta,p\right)$ or $\rho = \hat{\rho}\left(S_{\mathrm{A}},\Theta,p\right)$, potential density with respect to reference pressure p_{r} (e. g. 1000 dbar) can be easily evaluated as

$$\rho^{\theta}\left(S_{\mathrm{A}},t,p,p_{\mathrm{r}}\right) = \rho^{\Theta}\left(S_{\mathrm{A}},t,p,p_{\mathrm{r}}\right) = \bar{\rho}\left(S_{\mathrm{A}},\eta,\ p_{\mathrm{r}}\right) = \tilde{\rho}\left(S_{\mathrm{A}},\theta,\ p_{\mathrm{r}}\right) = \hat{\rho}\left(S_{\mathrm{A}},\Theta,\ p_{\mathrm{r}}\right), \qquad (3.4.3)$$

where we note that the potential temperature θ in the third expression is the potential temperature with respect to 0 dbar. Once the reference pressure is fixed, potential density is a function only of Absolute Salinity and Conservative Temperature (or equivalently, of Absolute Salinity and potential temperature). Note that it is equally correct to label potential density as ρ^{θ} or ρ^{Θ} (or indeed as ρ^{η}) because η, θ and Θ are constant during the isentropic and isohaline pressure change from p to p_{r}; that is, these variables posses the "potential" property of appendix A.9.

Following the discussion after Eqn. (2.8.2) above, potential density may also be expressed in terms of the pressure derivative of the expressions $h = \tilde{h}\left(S_{\mathrm{A}},\theta,p\right)$ and $h = \hat{h}\left(S_{\mathrm{A}},\Theta,p\right)$ for enthalpy as (see also appendix P)

$$\rho^{\theta}\left(S_{\mathrm{A}},t,p,p_{\mathrm{r}}\right) = \rho^{\Theta}\left(S_{\mathrm{A}},t,p,p_{\mathrm{r}}\right) = \left[\tilde{h}_P\left(S_{\mathrm{A}},\theta,\ p = p_{\mathrm{r}}\right)\right]^{-1} = \left[\hat{h}_P\left(S_{\mathrm{A}},\Theta,\ p = p_{\mathrm{r}}\right)\right]^{-1}. \qquad (3.4.4)$$