Notes on the function gsw_geo_strf_dyn_height

Notes made 21st September 2010

The code gsw_geo_strf_dyn_height uses the computationally efficient 25-term rational function expression of McDougall *et al.* (2010) for the specific volume of seawater $\hat{v}(S_A, \Theta, p)$ in terms of Absolute Salinity S_A, Conservative Temperature Θ and pressure p. The specific volume anomaly is defined with respect to $S_A = S_{SO} \equiv 35.165\,04 \text{ g kg}^{-1}$ and $\Theta = 0°\text{C}$ as

$$\hat{\delta}(S_A, \Theta, p) = \hat{v}(S_A, \Theta, p) - \hat{v}(S_{SO}, 0°\text{C}, p), \tag{1}$$

and the thermodynamic identity

$$h_P\big|_{S_A,\Theta} = \hat{h}_P = \hat{v}, \tag{2}$$

is used to calculate the dynamic height anomaly Ψ according to

$$\begin{aligned} \Psi &= -\int_{P_0}^{P} \hat{\delta}\left(S_A[p'], \Theta[p'], p'\right) dP' \\ &= -\int_{P_0}^{P} \hat{v}\left(S_A[p'], \Theta[p'], p'\right) dP' + \int_{P_0}^{P} \hat{v}\left(S_{SO}, \Theta = 0°\text{C}, p'\right) dP' \\ &= -\int_{P_0}^{P} \hat{v}\left(S_A[p'], \Theta[p'], p'\right) dP' + \hat{h}\left(S_{SO}, \Theta = 0°\text{C}, p\right). \end{aligned} \tag{3}$$

Note the lower limit of the pressure integral of $\hat{v}(S_{SO}, 0°\text{C}, p')$ is $\hat{h}(S_{SO}, \Theta = 0°\text{C}, 0\,\text{dbar})$ which is zero (being c_p^0 times $\Theta = 0°\text{C}$). Under the default setting, this function gsw_geo_strf_dyn_height evaluates the pressure integral of specific volume using S_A and Θ "interploted" with respect to pressure using a scheme based on the method of Reiniger and Ross (1968). Our method uses a weighted mean of (i) values obtained from linear interpolation of the two nearest data points, and (ii) a linear extrapolation of the pairs of data above and below. This method resembles the use of cubic splines, and was developed for the construction of the CSIRO Atlas of Regional Seas (Ridgway *et al*., 2002). If the option "linear" is chosen in gsw_geo_strf_dyn_height, the function interpolates Absolute Salinity and Conservative Temperature linearly with presure in the vertical between "bottles".

The vertical (pressure) integral in the last line of Eqn. (3) is done over vertical intervals no larger than $1\text{x}10^4$ Pa (1 dbar). If the "bottles" are spaced more than 1 dbar in the vertical, extra bottles are created before their specific volume is calculated and the integral performed. This is done to maintain high accuracy since specific volume is a nonlinear function of S_A, Θ and p.

References

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org See section 3.27 and appendix A.30.

McDougall T. J., D. R. Jackett, P. M. Barker, C. Roberts-Thomson, R. Feistel and R. W. Hallberg, 2010: A computationally efficient 25-term expression for the density of seawater in terms of Conservative Temperature, and related properties of seawater. To be submitted to *Ocean Science Discussions*.

Reiniger, R. F. and C. K. Ross, 1968: A method of interpolation with application to oceanographic data. Deep-Sea Res. 15, 185-193.

Ridgway K. R., J. R. Dunn, and J. L. Wilkin, 2002: Ocean interpolation by four-dimensional least squares -Application to the waters around Australia, *J. Atmos. Ocean. Tech.*, **19**, 1357-1375.

Here follows section 3.27 from the TEOS-10 Manual (IOC *et al*. (2010)).

3.27 Dynamic height anomaly

The dynamic height anomaly Ψ, given by the vertical integral

$$\Psi \;=\; -\int_{P_0}^{P} \delta\big(S_{\mathrm{A}}[p'],t[p'],p'\big)\,dP', \tag{3.27.1}$$

is the geostrophic streamfunction for the flow at pressure P with respect to the flow at the sea surface and δ is the specific volume anomaly. Thus the two-dimensional gradient of Ψ in the P pressure surface is simply related to the difference between the horizontal geostrophic velocity $\mathbf{v}$ at P and at the sea surface $\mathbf{v}_0$ according to

$$\mathbf{k}\times\nabla_P\Psi \;=\; f\mathbf{v} - f\mathbf{v}_0. \tag{3.27.2}$$

The definition Eqn. (3.27.1) of dynamic height anomaly applies to all choices of the reference values $\tilde{S}_{\mathrm{A}}$ and $\hat{t}$, $\tilde{\theta}$ or $\hat{\Theta}$ in the definition Eqns. (3.7.1 – 3.7.4) of the specific volume anomaly δ. Also, δ in Eqn. (3.27.1) can be replaced with specific volume v without affecting the isobaric gradient of the resulting streamfunction. That is, this substitution does not affect Eqn. (3.27.2) because the additional term is a function only of pressure. Traditionally it was important to use specific volume anomaly in preference to specific volume as it was more accurate with computer code which worked with single-precision variables. Since computers now regularly employ double-precision, this issue has been overcome and consequently either δ or v can be used in the integrand of Eqn. (3.27.1), so making it either the "dynamic height anomaly" or the "dynamic height". As in the case of Eqn. (3.24.2), so also the dynamic height anomaly Eqn. (3.27.1) has not assumed that the gravitational acceleration is constant and so Eqn. (3.27.2) applies even when the gravitational acceleration is taken to vary in the vertical.

The dynamic height anomaly Ψ should be quoted in units of $\mathrm{m^2\ s^{-2}}$. These are the units in which the GSW library (appendix N) outputs dynamic height anomaly in the function gsw_geo_strf_dyn_height. Note that the integration in Eqn. (3.27.1) of specific volume anomaly with pressure in dbar would yield dynamic height anomaly in units of $\mathrm{m^3\ kg^{-1}dbar}$, and the use of these units in Eqn. (3.27.2) would not give the resultant horizontal gradient in the usual units, being the product of the Coriolis parameter (units of $\mathrm{s^{-1}}$) and the velocity (units of $\mathrm{m\,s^{-1}}$). This is the reason why the pressure integration is done with pressure in Pa and dynamic height anomaly is output in $\mathrm{m^2\ s^{-2}}$.