

## Notes on the function, `gsw_CT_from_entropy(SA, entropy)`, which evaluates Conservative Temperature from specific entropy

This function, `gsw_CT_from_entropy`, finds Conservative Temperature for given values of Absolute Salinity and specific entropy,  $\Theta = \Theta(S_A, \eta)$ . The function amounts to the following simple calls to two other GSW functions,

```
pt = gsw_pt_from_entropy(SA, entropy);
CT = gsw_CT_from_pt(SA, pt);
```

Here follows appendix A.10 of the TEOS-10 Manual (IOC *et al.* (2010)).

### A.10 Proof that $\theta = \theta(S_A, \eta)$ and $\Theta = \Theta(S_A, \theta)$

Consider changes occurring at the sea surface, (specifically at  $p = 0$  dbar) where the temperature is the same as the potential temperature referenced to 0 dbar and the increment of pressure  $dp$  is zero. Regarding specific enthalpy  $h$  and chemical potential  $\mu$  to be functions of entropy  $\eta$  (in place of temperature  $t$ ), that is, considering the functional form of  $h$  and  $\mu$  to be  $h = \hat{h}(S_A, \eta, p)$  and  $\mu = \hat{\mu}(S_A, \eta, p)$ , it follows from the fundamental thermodynamic relation (Eqn. (A.7.1)) that

$$\hat{h}_\eta(S_A, \eta, 0) d\eta + \hat{h}_{S_A}(S_A, \eta, 0) dS_A = (T_0 + \theta) d\eta + \mu(S_A, \eta, 0) dS_A, \quad (\text{A.10.1})$$

which shows that specific entropy  $\eta$  is simply a function of Absolute Salinity  $S_A$  and potential temperature  $\theta$ , that is  $\eta = \eta(S_A, \theta)$ , with no separate dependence on pressure. It follows that  $\theta = \theta(S_A, \eta)$ .

Similarly, from the definition of potential enthalpy and Conservative Temperature in Eqns. (3.2.1) and (3.3.1), at  $p = 0$  dbar it can be seen that the fundamental thermodynamic relation (A.7.1) implies

$$c_p^0 d\Theta = (T_0 + \theta) d\eta + \tilde{\mu}(S_A, \theta, 0) dS_A. \quad (\text{A.10.2})$$

This shows that Conservative Temperature is also simply a function of Absolute Salinity and potential temperature,  $\Theta = \Theta(S_A, \theta)$ , with no separate dependence on pressure. It then follows that  $\Theta$  may also be expressed as a function of only  $S_A$  and  $\eta$ . It follows that  $\Theta$  has the “potential” property.