3.7 Specific volume anomaly

The specific volume anomaly δ is defined as the difference between the specific volume and a given function of pressure. Traditionally δ has been defined as

$$\delta\left(S_{\mathrm{A}}, t, p\right) = v\left(S_{\mathrm{A}}, t, p\right) - v\left(S_{\mathrm{SO}}, 0°\mathrm{C}, p\right) \tag{3.7.1}$$

(where the traditional value of Practical Salinity of 35 has been updated to an Absolute Salinity of $S_{\mathrm{SO}} = 35 u_{PS} = 35.16504\ \mathrm{g\,kg^{-1}}$ in the present formulation). Note that the second term, $v\left(S_{\mathrm{SO}}, 0°\mathrm{C}, p\right)$, is a function only of pressure. In order to have a surface of constant specific volume anomaly more accurately approximate neutral tangent planes (see section 3.11), it is advisable to replace the arguments S_{SO} and $0°\mathrm{C}$ with more general values $\widehat{\widehat{S}}_{\mathrm{A}}$ and $\widehat{\widehat{t}}$ that are carefully chosen (as say the median values of Absolute Salinity and temperature along the surface) so that the more general definition of specific volume anomaly is

$$\widehat{\widehat{\delta}}\left(S_{\mathrm{A}}, t, p\right) = v\left(S_{\mathrm{A}}, t, p\right) - v\left(\widehat{\widehat{S}}_{\mathrm{A}}, \widehat{\widehat{t}}, p\right) = g_P\left(S_{\mathrm{A}}, t, p\right) - g_P\left(\widehat{\widehat{S}}_{\mathrm{A}}, \widehat{\widehat{t}}, p\right). \tag{3.7.2}$$

The last terms in Eqns. (3.7.1) and (3.7.2) are simply functions of pressure and one has the freedom to choose any other function of pressure in its place and still retain the dynamical properties of specific volume anomaly. In particular, one can construct specific volume and enthalpy to be functions of Conservative Temperature (rather than *in situ* temperature) as $\hat{v}\left(S_{\mathrm{A}}, \Theta, p\right)$ and $\hat{h}\left(S_{\mathrm{A}}, \Theta, p\right)$ and write a slightly different definition of specific volume anomaly as

$$\tilde{\tilde{\delta}}\left(S_{\mathrm{A}}, \Theta, p\right) = \hat{v}\left(S_{\mathrm{A}}, \Theta, p\right) - \hat{v}\left(\tilde{\tilde{S}}_{\mathrm{A}}, \tilde{\tilde{\Theta}}, p\right) = \hat{h}_P\left(S_{\mathrm{A}}, \Theta, p\right) - \hat{h}_P\left(\tilde{\tilde{S}}_{\mathrm{A}}, \tilde{\tilde{\Theta}}, p\right). \tag{3.7.3}$$

The same can also be done with potential temperature so that in terms of the specific volume $\tilde{v}\left(S_{\mathrm{A}}, \theta, p\right)$ and enthalpy $\tilde{h}\left(S_{\mathrm{A}}, \theta, p\right)$ we can write another form of the specific volume anomaly as

$$\tilde{\tilde{\delta}}\left(S_{\mathrm{A}}, \theta, p\right) = \tilde{v}\left(S_{\mathrm{A}}, \theta, p\right) - \tilde{v}\left(\tilde{\tilde{S}}_{\mathrm{A}}, \tilde{\tilde{\theta}}, p\right) = \tilde{h}_P\left(S_{\mathrm{A}}, \theta, p\right) - \tilde{h}_P\left(\tilde{\tilde{S}}_{\mathrm{A}}, \tilde{\tilde{\theta}}, p\right). \tag{3.7.4}$$

These expressions exploit the fact that (see appendix A.11)

$$\left.\partial h/\partial P\right|_{S_{\mathrm{A}},\eta} = \left.\partial h/\partial P\right|_{S_{\mathrm{A}},\Theta} = \left.\partial h/\partial P\right|_{S_{\mathrm{A}},\theta} = v\,. \tag{3.7.5}$$