3.29 Cunningham geostrophic streamfunction

Cunningham (2000) and Alderson and Killworth (2005), following Saunders (1995) and Killworth (1986), suggested that a suitable streamfunction on a density surface in a compressible ocean would be the difference between the Bernoulli function $\mathscr{B}$ and potential enthalpy h^0. Since the kinetic energy per unit mass, $0.5\mathbf{u}\cdot\mathbf{u}$, is a tiny component of the Bernoulli function, it was ignored and Cunningham (2000) essentially proposed the streamfunction $\Pi + \Phi^0$ (see his equation (12)), where

$$\begin{aligned}\Pi &\equiv \mathscr{B} - h^0 - \tfrac{1}{2}\mathbf{u}\cdot\mathbf{u} - \Phi^0 \\ &= h - h^0 + \Phi - \Phi^0 \\ &= h(S_\mathrm{A},\Theta,p) - h(S_\mathrm{A},\Theta,0) - \int_{P_0}^{P} \hat{v}\left(S_\mathrm{A}(p'),\Theta(p'),p'\right) dP'.\end{aligned} \tag{3.29.1}$$

The last line of this equation has used the hydrostatic equation $P_z = -g\rho$ to express $\Phi \approx gz$ in terms of the vertical pressure integral of specific volume and the height of the sea surface where the geopotential is Φ^0.

The definition of potential enthalpy, Eqn. (3.2.1), is used to rewrite the last line of Eqn. (3.29.1), showing that Cunningham's Π is also equal to

$$\Pi = -\int_{P_0}^{P} \hat{v}\left(S_\mathrm{A}(p'),\Theta(p'),p'\right) - \hat{v}\left(S_\mathrm{A},\Theta,p'\right) dP'. \tag{3.29.2}$$

In this form it appears very similar to the expression, Eqn. (3.27.1), for dynamic height anomaly, the only difference being that in Eqn. (3.27.1) the pressure-independent values of Absolute Salinity and Conservative Temperature were S_SO and $0°\mathrm{C}$ whereas here they are the local values on the surface, S_A and Θ. While these local values of Absolute Salinity and Conservative Temperature are constant during the pressure integral in Eqn. (3.29.2), they do vary with latitude and longitude along any "density" surface.

The gradient of Π along the neutral tangent plane is

$$\nabla_n \Pi \approx \left\{ \tfrac{1}{\rho}\nabla_z P - \nabla\Phi_0 \right\} - \tfrac{1}{2}\rho^{-1} T_\mathrm{b}^{\Theta} \left(P - P_0\right)^2 \nabla_n \Theta, \tag{3.29.3}$$

(from McDougall and Klocker (2010)) so that the error in $\nabla_n\Pi$ in using Π as the geostrophic streamfunction is approximately $-\tfrac{1}{2}\rho^{-1}T_\mathrm{b}^{\Theta}\left(P - P_0\right)^2 \nabla_n\Theta$. When using the Cunningham streamfunction Π in a potential density surface, the error in $\nabla_\sigma\Pi$ is approximately $-\tfrac{1}{2}\rho^{-1}T_\mathrm{b}^{\Theta}\left(P - P_0\right)\left(2P_\mathrm{r} - P - P_0\right)\nabla_\sigma\Theta$. The Cunningham geostrophic streamfunction should be quoted in units of $\mathrm{m}^2\ \mathrm{s}^{-2}$ and is available in the GSW software library (appendix N) as the function gsw_geo_strf_Cunningham.