A.14 Advective and diffusive "heat" fluxes

In section 3.23 and appendices A.8 and A.13 the First Law of Thermodynamics is shown to be practically equivalent to the conservation equation (A.21.15) for Conservative Temperature Θ. We have emphasized that this means that the advection of "heat" is very accurately given as the advection of $c_p^0\Theta$. In this way $c_p^0\Theta$ can be regarded as the "heat content" per unit mass of seawater and the error involved with making this association is approximately 1% of the error in assuming that either $c_p^0\theta$ or $c_p\left(S_{\mathrm{A}},\theta,0\,\mathrm{dbar}\right)\theta$ is the "heat content" per unit mass of seawater (see also appendix A.21 for a discussion of this point).

The conservative form (A.21.15) implies that the turbulent diffusive flux of heat should be directed down the mean gradient of Conservative Temperature rather than down the mean gradient of potential temperature. In this appendix we quantify the difference between these mean temperature gradients.

Consider first the respective temperature gradients along the neutral tangent plane. From Eqn. (3.11.2) we find that

$$\left(\alpha^{\theta}/\beta^{\theta}\right)\nabla_n\theta \;=\; \nabla_n S_{\mathrm{A}} = \;\left(\alpha^{\Theta}/\beta^{\Theta}\right)\nabla_n\Theta, \qquad \text{(A.14.1)}$$

so that the epineutral gradients of θ and Θ are related by the ratios of their respective thermal expansion and saline contraction coefficients, namely

$$\nabla_n\theta \;=\; \frac{\left(\alpha^{\Theta}/\beta^{\Theta}\right)}{\left(\alpha^{\theta}/\beta^{\theta}\right)}\nabla_n\Theta. \qquad \text{(A.14.2)}$$

This proportionality factor between the parallel two-dimensional vectors $\nabla_n\theta$ and $\nabla_n\Theta$ is readily calculated and illustrated graphically. Before doing so we note two other equivalent expressions for this proportionality factor.

The epineutral gradients of θ, Θ and S_{A} are related by (using $\theta = \hat{\theta}\left(S_{\mathrm{A}},\Theta\right)$)

$$\nabla_n\theta \;=\; \hat{\theta}_{\Theta}\,\nabla_n\Theta \;+\; \hat{\theta}_{S_{\mathrm{A}}}\nabla_n S_{\mathrm{A}}\,, \qquad \text{(A.14.3)}$$

and using the neutral relationship $\nabla_n S_{\mathrm{A}} = \left(\alpha^{\Theta}/\beta^{\Theta}\right)\nabla_n\Theta$ we find

$$\nabla_n\theta \;=\; \left(\hat{\theta}_{\Theta} + \left[\alpha^{\Theta}/\beta^{\Theta}\right]\hat{\theta}_{S_{\mathrm{A}}}\right)\nabla_n\Theta. \qquad \text{(A.14.4)}$$

Also, in section 3.13 we found that $T_b^{\theta}\nabla_n\theta \;=\; T_b^{\Theta}\nabla_n\Theta$, so that we can write the equivalent expressions

$$\frac{\left|\nabla_n\theta\right|}{\left|\nabla_n\Theta\right|} \;=\; \frac{\left(\alpha^{\Theta}/\beta^{\Theta}\right)}{\left(\alpha^{\theta}/\beta^{\theta}\right)} \;=\; \frac{T_b^{\Theta}}{T_b^{\theta}} \;=\; \hat{\theta}_{\Theta} + \left[\alpha^{\Theta}/\beta^{\Theta}\right]\hat{\theta}_{S_{\mathrm{A}}}\,, \qquad \text{(A.14.5)}$$

and it can be shown that $\alpha^{\Theta}/\alpha^{\theta} \;=\; \hat{\theta}_{\Theta}$ and $\beta^{\theta}/\beta^{\Theta} \;=\; \left(1 + \left[\alpha^{\Theta}/\beta^{\Theta}\right]\hat{\theta}_{S_{\mathrm{A}}}/\hat{\theta}_{\Theta}\right)$, that is, $\beta^{\theta} = \beta^{\Theta} + \alpha^{\Theta}\,\hat{\theta}_{S_{\mathrm{A}}}/\hat{\theta}_{\Theta}$. The partial derivatives $\hat{\theta}_{\Theta}$ and $\hat{\theta}_{S_{\mathrm{A}}}$ in the last part of Eqn. (A.14.5) are both independent of pressure while $\alpha^{\Theta}/\beta^{\Theta}$ is a function of pressure. This ratio, Eqn. (A.14.5), of the epineutral gradients of θ and Θ is shown in Figure A.14.1 at $p = 0$, indicating that the epineutral gradient of potential temperature is sometimes more that 1% different to that of Conservative Temperature. This ratio $\left|\nabla_n\theta\right|/\left|\nabla_n\Theta\right|$ is only a weak function of pressure. This ratio, $\left|\nabla_n\theta\right|/\left|\nabla_n\Theta\right|$ (i.e. Eqn. (A.14.5)), is available in the GSW computer software library as function gsw_ntp_pt_to_CT_gradient.

Similarly to Eqn. (A.14.3), the vertical gradients are related by

$$\theta_z \;=\; \hat{\theta}_{\Theta}\,\Theta_z \;+\; \hat{\theta}_{S_{\mathrm{A}}}S_{\mathrm{A}_z}\,, \qquad \text{(A.14.6)}$$

and using the definition, Eqn. (3.15.1), of the stability ratio we find that

$$\frac{\theta_z}{\Theta_z} = \hat{\theta}_{\Theta} + R_\rho^{-1}\left[\alpha^{\Theta}/\beta^{\Theta}\right]\hat{\theta}_{S_\mathrm{A}} . \qquad \text{(A.14.7)}$$

For values of the stability ratio R_ρ close to unity, the ratio θ_z/Θ_z is close to the values of $|\nabla_n\theta|/|\nabla_n\Theta|$ shown in Figure A.14.1. For other values of R_ρ, Eqn. (A.14.7) can be calculated and plotted.

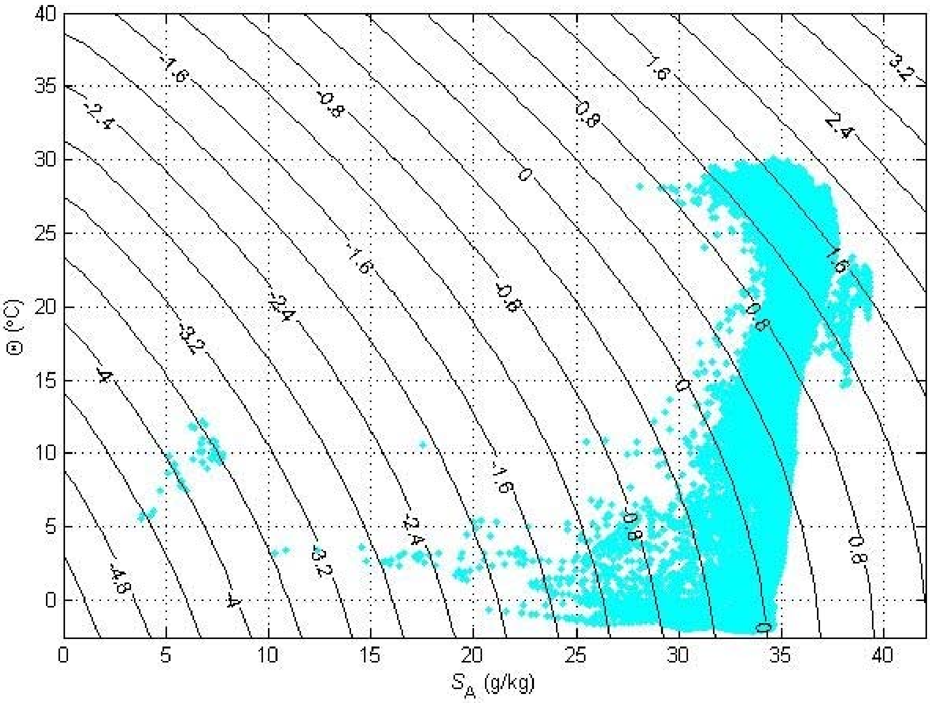

Figure A.14.1. Contours of $\left(|\nabla_n\theta|/|\nabla_n\Theta| - 1\right) \times 100\%$ at $p = 0$, showing the percentage difference between the epineutral gradients of θ and Θ. The blue dots are from the ocean atlas of Gouretski and Koltermann (2004) at $p = 0$.

As noted in section 3.8 the dianeutral advection of thermobaricity is the same when quantified in terms of potential temperature as when done in terms of Conservative Temperature. The same is not true of the dianeutral velocity caused by cabbeling. The ratio of the cabbeling dianeutral velocity calculated using potential temperature to that using Conservative Temperature is given by $\left(C_\mathrm{b}^{\theta}\nabla_n\theta\cdot\nabla_n\theta\right)/\left(C_\mathrm{b}^{\Theta}\nabla_n\Theta\cdot\nabla_n\Theta\right)$ (see section 3.9) which can be expressed as

$$\frac{C_\mathrm{b}^{\theta}|\nabla_n\theta|^2}{C_\mathrm{b}^{\Theta}|\nabla_n\Theta|^2} = \frac{C_\mathrm{b}^{\theta}}{C_\mathrm{b}^{\Theta}}\frac{\left(\alpha^{\Theta}/\beta^{\Theta}\right)^2}{\left(\alpha^{\theta}/\beta^{\theta}\right)^2} = \frac{C_\mathrm{b}^{\theta}}{C_\mathrm{b}^{\Theta}}\left(\frac{T_\mathrm{b}^{\Theta}}{T_\mathrm{b}^{\theta}}\right)^2 = \frac{C_\mathrm{b}^{\theta}}{C_\mathrm{b}^{\Theta}}\left(\hat{\theta}_{\Theta} + \left[\alpha^{\Theta}/\beta^{\Theta}\right]\hat{\theta}_{S_\mathrm{A}}\right)^2 , \qquad \text{(A.14.8)}$$

and this is contoured in Fig. A.14.2. While the ratio of Eqn. (A.14.8) is not exactly unity, it varies relatively little in the oceanographic range, indicating that the dianeutral advection due to cabbeling estimated using θ or Θ are within half a percent of each other at $p = 0$.

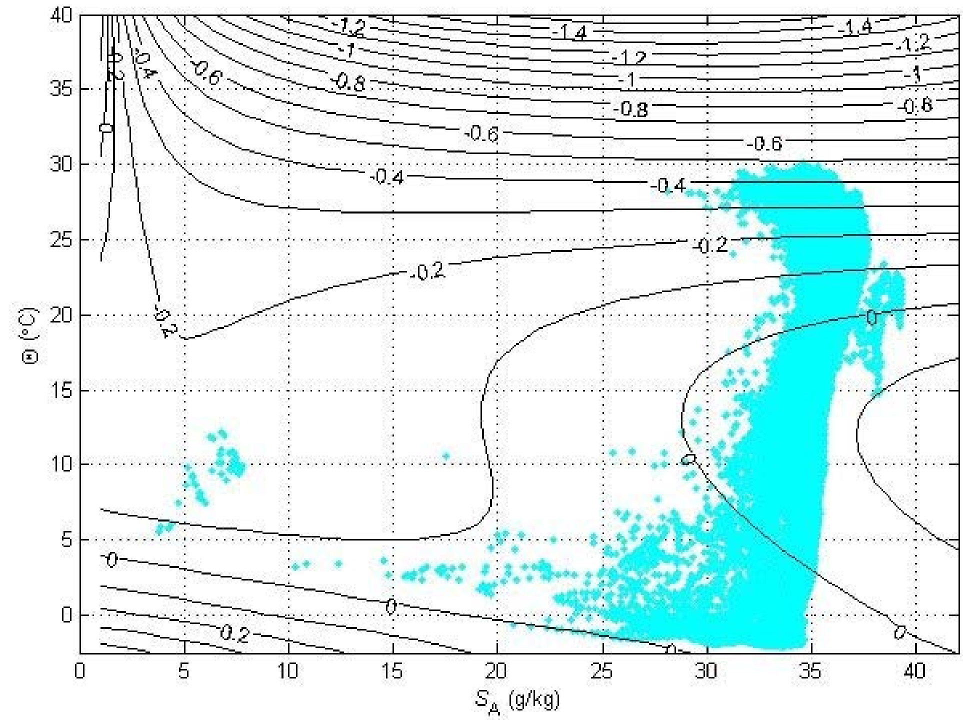

Figure A.14.2. Contours of the percentage difference of $\left(C_b^\theta \left|\nabla_n \theta\right|^2\right) \Big/ \left(C_b^\Theta \left|\nabla_n \Theta\right|^2\right)$ from unity at $p = 0$ dbar.