3.28 Montgomery geostrophic streamfunction

The Montgomery "acceleration potential" π defined by

$$\pi \;=\; \left(P - P_0\right)\delta \;-\; \int_{P_0}^{P} \delta\left(S_{\mathrm{A}}\left[p'\right], t\left[p'\right], p'\right) dP' \qquad (3.28.1)$$

is the geostrophic streamfunction for the flow in the specific volume anomaly surface $\delta\left(S_{\mathrm{A}}, t, p\right) = \delta_1$ relative to the flow at $P = P_0$ (that is, at $p = 0\,\mathrm{dbar}$). Thus the two-dimensional gradient of π in the δ_1 specific volume anomaly surface is simply related to the difference between the horizontal geostrophic velocity $\mathbf{v}$ in the $\delta = \delta_1$ surface and at the sea surface $\mathbf{v}_0$ according to

$$\mathbf{k}\times\nabla_{\delta_1}\pi \;=\; f\mathbf{v} - f\mathbf{v}_0 \qquad \text{or} \qquad \nabla_{\delta_1}\pi \;=\; -\mathbf{k}\times\left(f\mathbf{v} - f\mathbf{v}_0\right). \qquad (3.28.2)$$

The definition, Eqn. (3.28.1), of the Montgomery geostrophic streamfunction applies to all choices of the reference values $\hat{\hat{S}}_{\mathrm{A}}$ and $\hat{\hat{t}}$ in the definition, Eqn. (3.7.2), of the specific volume anomaly δ. By carefully choosing these reference values the specific volume anomaly surface can be made to closely approximate the neutral tangent plane (McDougall and Jackett (2007)).

It is not uncommon to read of authors using the Montgomery geostrophic streamfunction, Eqn (3.28.1), as a geostrophic streamfunction in surfaces other than specific volume anomaly surfaces. This incurs errors that should be recognized. For example, the gradient of the Montgomery geostrophic streamfunction, Eqn. (3.28.1), in a neutral tangent plane becomes (instead of Eqn. (3.28.2) in the $\delta = \delta_1$ surface)

$$\nabla_n\pi \;=\; -\mathbf{k}\times\left(f\mathbf{v} - f\mathbf{v}_0\right) + \left(P - P_0\right)\nabla_n\delta\,, \qquad (3.28.3)$$

where the last term represents an error arising from using the Montgomery streamfunction in a surface other than the surface for which it was derived.

Zhang and Hogg (1992) subtracted an arbitrary pressure offset, $\left(\bar{P} - P_0\right)$, from $\left(P - P_0\right)$ in the first term in Eqn. (3.28.1), so defining the modified Montgomery streamfunction

$$\pi^{\text{Z-H}} \;=\; \left(P - \bar{P}\right)\delta \;-\; \int_{P_0}^{P} \delta\left(S_{\mathrm{A}}\left[p'\right], t\left[p'\right], p'\right) dP'. \qquad (3.28.4)$$

The gradient of $\pi^{\text{Z-H}}$ in a neutral tangent plane becomes

$$\nabla_n\pi^{\text{Z-H}} \;=\; -\mathbf{k}\times\left(f\mathbf{v} - f\mathbf{v}_0\right) \;+\; \left(P - \bar{P}\right)\nabla_n\delta\,, \qquad (3.28.5)$$

where the last term can be made significantly smaller than the corresponding term in Eqn. (3.28.3) by choosing the constant pressure $\bar{P}$ to be close to the average pressure on the surface.

This term can be further minimized by suitably choosing the constant reference values $\tilde{\tilde{S}}_{\mathrm{A}}$ and $\tilde{\tilde{\Theta}}$ in the definition, Eqn. (3.7.3), of specific volume anomaly $\tilde{\tilde{\delta}}$ so that this surface more closely approximates the neutral tangent plane (McDougall (1989)). This improvement is available because it can be shown that

$$\rho\nabla_n\tilde{\tilde{\delta}} \;=\; -\left[\kappa\left(S_{\mathrm{A}}, \Theta, p\right) - \kappa\left(\tilde{\tilde{S}}_{\mathrm{A}}, \tilde{\tilde{\Theta}}, p\right)\right]\nabla_n P \;\approx\; T_b^{\Theta}\left(\Theta - \tilde{\tilde{\Theta}}\right)\nabla_n P. \qquad (3.28.6)$$

The last term in Eqn. (3.28.5) is then approximately

$$\left(P - \bar{P}\right)\nabla_n\tilde{\tilde{\delta}} \;\approx\; \tfrac{1}{2}\rho^{-1}T_b^{\Theta}\left(\Theta - \tilde{\tilde{\Theta}}\right)\nabla_n\left(P - \bar{P}\right)^2 \qquad (3.28.7)$$

and hence suitable choices of $\bar{P}$, $\tilde{\tilde{S}}_{\mathrm{A}}$ and $\tilde{\tilde{\Theta}}$ can reduce the last term in Eqn. (3.28.5) that represents the error in interpreting the Montgomery geostrophic streamfunction, Eqn. (3.28.4), as the geostrophic streamfunction in a surface that is more neutral than a specific volume anomaly surface.

The Montgomery geostrophic streamfunction should be quoted in units of $m^2 \ s^{-2}$. These are the units in which the GSW library (appendix N) outputs the Montgomery geostrophic streamfunction in the function gsw_geo_strf_Montgomery.