2.7 Specific volume

The specific volume of seawater v is given by the pressure derivative of the Gibbs function at constant Absolute Salinity S_A and *in situ* temperature t, that is

$$v = v(S_A, t, p) = g_P = \partial g/\partial P|_{S_A,T}\,. \tag{2.7.1}$$

Notice that specific volume is a function of Absolute Salinity S_A rather than of Reference Salinity S_R or Practical Salinity S_P. The importance of this point is discussed in section 2.8. When derivatives are taken with respect to *in situ* temperature, or at constant *in situ* temperature, the symbol t is avoided as it can be confused with the same symbol for time. Rather, we use T in place of t in the expressions for these derivatives.

For many theoretical and modeling purposes in oceanography it is convenient to regard the independent temperature variable to be potential temperature θ or Conservative Temperature Θ rather than *in situ* temperature t. We note here that the specific volume is equal to the pressure derivative of specific enthalpy at fixed Absolute Salinity when any one of η, θ or Θ is also held constant, as follows (from appendix A.11)

$$\partial h/\partial P|_{S_A,\eta} = \partial h/\partial P|_{S_A,\Theta} = \partial h/\partial P|_{S_A,\theta} = v\,. \tag{2.7.2}$$

The use of P in these equations emphasizes that it must be in Pa not dbar. Specific volume v has units of $\mathrm{m^3\ kg^{-1}}$ in both the SIA and GSW computer libraries.

2.8 Density

The density of seawater ρ is the reciprocal of the specific volume. It is given by the reciprocal of the pressure derivative of the Gibbs function at constant Absolute Salinity S_A and *in situ* temperature t, that is

$$\rho = \rho(S_A, t, p) = (g_P)^{-1} = \left(\partial g/\partial P|_{S_A,T}\right)^{-1}\,. \tag{2.8.1}$$

Notice that density is a function of Absolute Salinity S_A rather than of Reference Salinity S_R or Practical Salinity S_P. This is an extremely important point because Absolute Salinity S_A in units of $\mathrm{g\ kg^{-1}}$ is numerically greater than Practical Salinity by between $0.165\ \mathrm{g\ kg^{-1}}$ and $0.195\ \mathrm{g\ kg^{-1}}$ in the open ocean so that if Practical Salinity were inadvertently used as the salinity argument for the density algorithm, a significant density error of between $0.12\ \mathrm{kg\ m^{-3}}$ and $0.15\ \mathrm{kg\ m^{-3}}$ would result.

For many theoretical and modeling purposes in oceanography it is convenient to regard density to be a function of potential temperature θ or Conservative Temperature Θ rather than of *in situ* temperature t. That is, it is convenient to form the following two functional forms of density,

$$\rho = \tilde{\rho}(S_A, \theta, p) = \hat{\rho}(S_A, \Theta, p), \tag{2.8.2}$$

where θ and Θ are respectively potential temperature and Conservative Temperature, both referenced to $p_r = 0$ dbar. We will adopt the convention (see Table L.2 in appendix L) that when enthalpy h, specific volume v or density ρ are taken to be functions of potential temperature they attract an over-tilde as in $\tilde{v}$ or $\tilde{\rho}$, and when they are taken to be functions of Conservative Temperature they attract a caret as in $\hat{v}$ and $\hat{\rho}$. With this convention, expressions involving partial derivatives such as (2.7.2) can be written more compactly as (from appendix A.11)

$$\breve{h}_P = \tilde{h}_P = \hat{h}_P = v = \rho^{-1} \tag{2.8.3}$$

since the other variables are taken to be constant during the partial differentiation. Appendix P lists expressions for many thermodynamic variables in terms of the thermodynamic potentials

$$h = \hat{h}(S_A, \eta, p), \; h = \tilde{h}(S_A, \theta, p) \text{ and } h = \hat{h}(S_A, \Theta, p). \tag{2.8.4}$$

Density ρ has units of $\mathrm{kg\ m^{-3}}$ in both the SIA and GSW computer libraries.

Computationally efficient expressions for $\hat{\rho}(S_A, \Theta, p)$ and $\tilde{\rho}(S_A, \theta, p)$ involving 25 coefficients are available (McDougall *et al.* (2010b)) and are described in appendix A.30 and appendix K. These expressions can be integrated with respect to pressure to provide closed expressions for $\hat{h}(S_A, \Theta, p)$ and $\tilde{h}(S_A, \theta, p)$ (see Eqn. (A.30.6)).