## 3.17 Property gradients along potential density surfaces

The two-dimensional gradient of a scalar $\varphi$ along a potential density surface $\nabla_\sigma \varphi$ is related to the corresponding gradient in the neutral tangent plane $\nabla_n \varphi$ by

$$\nabla_\sigma \varphi \;=\; \nabla_n \varphi + \frac{\varphi_z}{\Theta_z} \frac{R_\rho \left[ r-1 \right]}{\left[ R_\rho - r \right]} \nabla_n \Theta \tag{3.17.1}$$

(from McDougall (1987a)), where $r$ is the ratio of the slope on the $S_\mathrm{A} - \Theta$ diagram of an isoline of potential density with reference pressure $p_\mathrm{r}$ to the slope of a potential density surface with reference pressure $p$, and is defined by

$$r \;=\; \frac{\alpha^\Theta \left( S_\mathrm{A}, \Theta, p \right) \big/ \beta^\Theta \left( S_\mathrm{A}, \Theta, p \right)}{\alpha^\Theta \left( S_\mathrm{A}, \Theta, p_\mathrm{r} \right) \big/ \beta^\Theta \left( S_\mathrm{A}, \Theta, p_\mathrm{r} \right)} \;. \tag{3.17.2}$$

Substituting $\varphi = \Theta$ into (3.17.1) gives the following relation between the (parallel) isopycnal and epineutral gradients of $\Theta$

$$\nabla_\sigma \Theta \;=\; \frac{r \left[ R_\rho - 1 \right]}{\left[ R_\rho - r \right]} \nabla_n \Theta \;=\; G^\Theta \nabla_n \Theta \tag{3.17.3}$$

where the "isopycnal temperature gradient ratio"

$$G^\Theta \;\equiv\; \frac{\left[ R_\rho - 1 \right]}{\left[ R_\rho / r - 1 \right]} \tag{3.17.4}$$

has been defined as a shorthand expression for future use. This ratio $G^\Theta$ is available in the GSW software library from the algorithm **gsw_isopycnal_vs_ntp_CT_ratio_CT25**, while the ratio $r$ of Eqn. (3.17.2) is available there as **gsw_isopycnal_slope_ratio_CT25**. Substituting $\varphi = S_\mathrm{A}$ into Eqn. (3.17.1) gives the following relation between the (parallel) isopycnal and epineutral gradients of $S_\mathrm{A}$

$$\nabla_\sigma S_\mathrm{A} \;=\; \frac{\left[ R_\rho - 1 \right]}{\left[ R_\rho - r \right]} \nabla_n S_\mathrm{A} \;=\; \frac{G^\Theta}{r} \nabla_n S_\mathrm{A} . \tag{3.17.5}$$