

Notes on the function `gsw_geo_strf_McD_Klocker_pc`

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In density-coordinate forward ocean models, the variables are interpreted as being piecewise constant in the vertical, and this function, `gsw_geo_strf_McD_Klocker_pc`, calculates the McDougall-Klocker geostrophic streamfunction taking this piecewise-constant nature of Absolute Salinity and Conservative Temperature into account. The code uses the computationally efficient 25-term rational function expression of McDougall *et al.* (2010) for the specific volume of seawater $\hat{v}(S_A, \Theta, p)$ in terms of Absolute Salinity S_A , Conservative Temperature Θ and pressure p .

In density-coordinate models (often called “isopycnal models”) the “thickness” of the i th layer, $p^{i+1} - p^i$, varies with latitude, longitude and time. In these layered models the “thicknesses” of the least dense layers are often zero at many locations in the ocean, because these density surfaces have outcropped.

The McDougall-Klocker geostrophic streamfunction is defined by

$$\varphi^n(S_A, \Theta, p) = \frac{1}{2}(P - \tilde{P})\tilde{\delta}(S_A, \Theta, p) - \frac{1}{12}\rho^{-1}T_b^\Theta(\Theta - \tilde{\Theta})(P - \tilde{P})^2 - \int_{P_0}^P \tilde{\delta} dP', \quad (1)$$

where $\rho^{-1}T_b^\Theta$ is taken to be the constant value $2.7 \times 10^{-15} \text{ K}^{-1}(\text{Pa})^{-2} \text{ m}^2 \text{ s}^{-2}$. This geostrophic streamfunction is for use in approximately neutral surfaces, such as an ω -surface of Klocker *et al.* (2009a,b) or a Neutral Density (γ^n) surface of Jackett and McDougall (1997) or a suitably referenced potential density surface. A suitable reference seawater parcel $(\tilde{S}_A, \tilde{\Theta}, \tilde{p})$ is selected for each approximately neutral surface that one is considering, and the specific volume anomaly $\tilde{\delta}$ is defined as

$$\tilde{\delta}(S_A, \Theta, p) = \hat{v}(S_A, \Theta, p) - \hat{v}(\tilde{S}_A, \tilde{\Theta}, \tilde{p}). \quad (2)$$

In this function, `gsw_geo_strf_McD_Klocker_pc`, the reference values $(\tilde{S}_A, \tilde{\Theta}, \tilde{p})$ are found by interpolation down a single reference cast which includes these three variables as well as values of Neutral Density γ^n and also of σ_2 . This reference cast is interpolated with respect to γ^n , to the value of γ^n which is supplied by the user of this function, with one value of γ^n for each “density” surface that is selected. If this function is being used with a series of approximately neutral surfaces which are neither Neutral Density surfaces nor ω -surfaces, then a value of γ^n needs to be assigned to each of the user’s surfaces. This could be achieved, for example, by the user labeling a vertical profile of her/his data from the mid equatorial Pacific with Neutral Density and associating these values of Neutral Density to the whole of the user’s layers; with one value of γ^n for each selected approximately neutral surface. As an alternative to having to label each layer with Neutral Density, we have also allowed for the option of having the reference data set to be interpolated with respect to σ_2 , the potential density with respect to 2000 dbar. In this case, the user needs to supply the σ_2 value for each layer on which the McDougall-Klocker streamfunction is required.

The last term in Eqn. (1) is obtained via a call to `gsw_geo_strf_dyn_height_pc`. This function returns the dynamic height anomaly with respect to $S_A = S_{SO} \equiv 35.16504 \text{ g kg}^{-1}$ and $\Theta = 0^\circ\text{C}$. In order to find the last term in Eqn. (1) we add the following terms to the output of `gsw_geo_strf_dyn_height_pc`,

$$\hat{h}(\tilde{S}_A, \tilde{\Theta}, \frac{1}{2}[p^{n+1} + p^n]) - c_p^0 \tilde{\Theta} - \hat{h}(S_{SO}, \Theta = 0^\circ\text{C}, \frac{1}{2}[p^{n+1} + p^n]), \quad (3)$$

where $\frac{1}{2}[p^{n+1} + p^n]$ is the pressure at the mid-pressure of the n th layer. The first term in Eqn. (3) is obtained from a call to `gsw_enthalpy_CT25` and the last term via a call to `gsw_enthalpy_SSO_0_CT25` which is simply a streamlined version of `gsw_enthalpy_CT25`.

The pressure input to the function `gsw_geo_strf_McD_Klocker_pc` is the matrix $\text{delta_p} = p^{i+1} - p^i$ and the code can handle zero values of this layer “thickness” in several layers (which occurs when density surface outcrop). The function calculates the pressures p_{mid} at the mid-pressure of each layer by summing the delta_p of the shallower layers and adding half of $\text{delta_p} = p^{i+1} - p^i$ at the i th layer (this summation is performed by `gsw_geo_strf_dyn_height_pc` and returned to the present function).

In the call, `gsw_geo_strf_McD_Klocker_pc(SA,CT,delta_p,gamma_n,A,'gn')`, the user supplies the Absolute Salinity SA, conservative Temperature CT and pressure thickness delta_p of many layers, with the first such layer being bounded by the sea surface. The user can request the McDougall-Klocker geostrophic streamfunction on each layer, or he/she can specify the indices of the layers on which the streamfunction is requested in the vector A. For example, $A = [6:12]$ implies that the user wants the streamfunction on layers 6 to 12 inclusive, and the vector gamma_n must be consistent with this choice, so that in this case, gamma_n must be 7 long. The last argument of the calling sequence to this function must be either ‘gn’ or ‘s2’ which tells the code whether the interpolating variable for the reference cast is γ^n or σ_2 .

A remark on the use of the McDougall-Klocker geostrophic streamfunction in layered ocean models

Section 8 of McDougall and Klocker (2010) discusses the use of the geostrophic streamfunction given by Eqn. (1) in density-coordinate ocean models. Here we expand a little on this discussion.

If the flow in the ocean is geostrophic then the difference between the horizontal velocity at pressure P and that at the sea surface is given by

$$-\mathbf{k} \times (f\mathbf{v} - f\mathbf{v}_0) = \left\{ \frac{1}{\rho} \nabla_z P - \nabla \Phi_0 \right\}, \quad (4)$$

where Φ_0 is the geopotential at the sea surface, and the use of upper case P for pressure serves to remind us that it needs to be in Pa in these expressions. From Eqn. (67) of McDougall and Klocker (2010) we see that $\left\{ \frac{1}{\rho} \nabla_z P - \nabla \Phi_0 \right\}$ can be expressed in terms of the two-dimensional gradient of φ^n (of Eqn. (1) above) in any general surface by

$$\left\{ \frac{1}{\rho} \nabla_z P - \nabla \Phi_0 \right\} = \nabla_\sigma \left[\varphi^n + \frac{1}{12} \rho^{-1} T_b^\Theta \left(\Theta - \tilde{\Theta} \right) \left(P - \tilde{P} \right)^2 \right] - \frac{1}{2} \left(P - \tilde{P} \right) \nabla_\sigma \tilde{\delta} + \frac{1}{2} \tilde{\delta} \nabla_\sigma P. \quad (5)$$

This equation demonstrates that in this application to a forward ocean model where all terms on the right-hand side of Eqn. (5) are evaluated during a model run, there is no point including the

$$- \frac{1}{12} \rho^{-1} T_b^\Theta \left(\Theta - \tilde{\Theta} \right) \left(P - \tilde{P} \right)^2 \quad (6)$$

term in the calculation of the geostrophic streamfunction (1) since it is subtracted from this streamfunction when it is used in Eqn. (5) in the forward model.

Hence, if this suggestion were adopted in a layered ocean model, the geostrophic streamfunction would be

$$\varphi(S_A, \Theta, p) = \frac{1}{2} \left(P - \tilde{P} \right) \tilde{\delta}(S_A, \Theta, p) - \int_{P_0}^P \tilde{\delta} dP' \quad (7)$$

and the desired horizontal pressure gradient term would be calculated in the forward ocean model using the right-hand side of the following equation,

$$\left\{ \frac{1}{\rho} \nabla_z P - \nabla \Phi_0 \right\} = \nabla_\sigma \varphi - \frac{1}{2} \left(P - \tilde{P} \right) \nabla_\sigma \tilde{\delta} + \frac{1}{2} \tilde{\delta} \nabla_\sigma P. \quad (8)$$

References

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Here follows section 3.30 from the TEOS-10 Manual (IOC *et al.* (2010)).

3.30 Geostrophic streamfunction in an approximately neutral surface

In order to evaluate a relatively accurate expression for the geostrophic streamfunction in an approximately neutral surface (such as an ω -surface of Klocker *et al.* (2009a,b) or a Neutral Density surface of Jackett and McDougall (1997)) a suitable reference seawater parcel $(\tilde{S}_A, \tilde{\Theta}, \tilde{p})$ is selected from the approximately neutral surface that one is considering, and the specific volume anomaly $\tilde{\delta}$ is defined as in (3.7.3) above. The approximate geostrophic streamfunction is given by (from McDougall and Klocker (2010))

$$\varphi^n(S_A, \Theta, p) = \frac{1}{2}(P - \tilde{P})\tilde{\delta}(S_A, \Theta, p) - \frac{1}{12}\rho^{-1}T_b^\Theta(\Theta - \tilde{\Theta})(P - \tilde{P})^2 - \int_{P_0}^P \tilde{\delta} dP'. \quad (3.30.1)$$

This expression is very accurate when the variation of conservative temperature with pressure along the approximately neutral surface is either linear or quadratic. That is, in these situations $\nabla_n \varphi^n \approx \frac{1}{\rho} \nabla_z P - \nabla \Phi_0 = -\mathbf{k} \times (f\mathbf{v} - f\mathbf{v}_0)$ to a very good approximation. In Eqn. (3.30.1) $\rho^{-1}T_b^\Theta$ is taken to be the constant value $2.7 \times 10^{-15} \text{K}^{-1}(\text{Pa})^{-2} \text{m}^2 \text{s}^{-2}$. This McDougall-Klocker geostrophic streamfunction is available from the GSW software as the function `gsw_geo_strf_McD_Klocker`.