Notes on the function, gsw_entropy_from_CT(SA, CT), which evaluates specific entropy from Conservative Temperature

This function, **gsw_entropy_from_CT**, finds $\eta = \eta(S_A, \Theta)$, specific entropy as a function of Absolute Salinity and Conservative Temperature. This is done by first evaluating potential temperature θ (with reference pressure $p_r = 0\ \text{dbar}$) from the function **gsw_pt_from_CT** and then calling the temperature derivative of the Gibbs function as follows,

$$\eta \;=\; \tilde{\eta}(S_A, \theta) \;=\; -g_T(S_A, \theta, p=0). \qquad (1)$$

Here follows appendix A.10 of the TEOS-10 Manual (IOC *et al*. (2010)).

A.10 Proof that $\theta = \theta(S_A, \eta)$ and $\Theta = \Theta(S_A, \theta)$

Consider changes occurring at the sea surface, (specifically at $p = 0$ dbar) where the temperature is the same as the potential temperature referenced to 0 dbar and the increment of pressure dp is zero. Regarding specific enthalpy h and chemical potential μ to be functions of entropy η (in place of temperature t), that is, considering the functional form of h and μ to be $h = \hat{h}(S_A, \eta, p)$ and $\mu = \hat{\mu}(S_A, \eta, p)$, it follows from the fundamental thermodynamic relation (Eqn. (A.7.1)) that

$$\hat{h}_\eta(S_A, \eta, 0)\,\mathrm{d}\eta + \hat{h}_{S_A}(S_A, \eta, 0)\,\mathrm{d}S_A \;=\; (T_0 + \theta)\mathrm{d}\eta + \mu(S_A, \eta, 0)\,\mathrm{d}S_A, \qquad (A.10.1)$$

which shows that specific entropy η is simply a function of Absolute Salinity S_A and potential temperature θ, that is $\eta = \eta(S_A, \theta)$, with no separate dependence on pressure. It follows that $\theta = \theta(S_A, \eta)$.

Similarly, from the definition of potential enthalpy and Conservative Temperature in Eqns. (3.2.1) and (3.3.1), at $p = 0$ dbar it can be seen that the fundamental thermodynamic relation (A.7.1) implies

$$c_p^0\,\mathrm{d}\Theta \;=\; (T_0 + \theta)\mathrm{d}\eta + \tilde{\mu}(S_A, \theta, 0)\,\mathrm{d}S_A. \qquad (A.10.2)$$

This shows that Conservative Temperature is also simply a function of Absolute Salinity and potential temperature, $\Theta = \Theta(S_A, \theta)$, with no separate dependence on pressure. It then follows that Θ may also be expressed as a function of only S_A and η. It follows that Θ has the "potential" property.