Notes on the function gsw_geo_strf_McD_Klocker

Notes made 23rd September 2010

The McDougall-Klocker geostrophic streamfunction is defined by

$$\varphi^{\mathrm{n}}\left(S_{\mathrm{A}},\Theta,p\right) \;=\; \tfrac{1}{2}\Big(P-\tilde{\tilde{P}}\Big)\tilde{\tilde{\delta}}\left(S_{\mathrm{A}},\Theta,p\right) \;-\; \tfrac{1}{12}\rho^{-1}T_{\mathrm{b}}^{\Theta}\Big(\Theta-\tilde{\tilde{\Theta}}\Big)\Big(P-\tilde{\tilde{P}}\Big)^{2} \;-\; \int_{P_0}^{P}\tilde{\tilde{\delta}}\;dP'\;, \qquad (1)$$

where $\rho^{-1}T_{\mathrm{b}}^{\Theta}$ is taken to be the constant value $2.7x10^{-15}\,\mathrm{K}^{-1}(\mathrm{Pa})^{-2}\mathrm{m}^{2}\mathrm{s}^{-2}$. The code uses the computationally efficient 25-term rational function expression of McDougall *et al*. (2010) for the specific volume of seawater $\hat{v}\left(S_{\mathrm{A}},\Theta,p\right)$ in terms of Absolute Salinity S_{A}, Conservative Temperature Θ and pressure p.

The McDougall-Klocker geostrophic streamfunction is for use in approximately neutral surfaces, such as an ω-surface of Klocker *et al.* (2009a,b) or a Neutral Density (γ^{n}) surface of Jackett and McDougall (1997)) or a suitably referenced potential density surface. A suitable reference seawater parcel $\Big(\tilde{\tilde{S}}_{\mathrm{A}},\tilde{\tilde{\Theta}},\tilde{\tilde{p}}\Big)$ is selected for each approximately neutral surface that one is considering, and the specific volume anomaly $\tilde{\tilde{\delta}}$ is defined as

$$\tilde{\tilde{\delta}}\left(S_{\mathrm{A}},\Theta,p\right) \;=\; \hat{v}\left(S_{\mathrm{A}},\Theta,p\right) \;-\; \hat{v}\Big(\tilde{\tilde{S}}_{\mathrm{A}},\tilde{\tilde{\Theta}},p\Big). \qquad (2)$$

In this function, gsw_geo_strf_McD_Klocker, the reference values $\Big(\tilde{\tilde{S}}_{\mathrm{A}},\tilde{\tilde{\Theta}},\tilde{\tilde{p}}\Big)$ are found by interpolation down a single reference cast which includes these three variables as well as values of Neutral Density γ^{n} and also of σ_2. This reference cast is interpolated with respect to γ^{n}, to the value of γ^{n} which is supplied by the user of this function, with one value of γ^{n} for each "density" surface that is selected. If this function is being used with a series of approximately neutral surfaces which are neither Neutral Density surfaces nor ω-surfaces, then a value of γ^{n} needs to be assigned to each of the user's surfaces. This could be achieved, for example, by the user labeling a vertical profile of her/his data from the mid equatorial Pacific with Neutral Density and associating these values of Neutral Density to the whole of the user's layers; with one value of γ^{n} for each selected approximately neutral surface. As an alternative to having to label each layer with Neutral Density, we have also allowed for the option of having the reference data set to be interpolated with respect to σ_2, the potential density with respect to 2000 dbar. In this case, the user needs to supply the σ_2 value for each layer on which the McDougall-Klocker streamfunction is required.

The last term in Eqn. (1) is obtained via a call to gsw_geo_strf_dyn_height. This function returns the dynamic height anomaly with respect to $S_{\mathrm{A}} = S_{\mathrm{SO}} \equiv 35.165\,04\ \mathrm{g\ kg}^{-1}$ and $\Theta = 0°\mathrm{C}$. In order to find the last term in Eqn. (1) we add the following terms to the output of gsw_geo_strf_dyn_height,

$$\hat{h}\Big(\tilde{\tilde{S}}_{\mathrm{A}},\tilde{\tilde{\Theta}},\,p\Big) \;-\; c_p^0\tilde{\tilde{\Theta}} \;-\; \hat{h}\left(S_{\mathrm{SO}},\Theta=0°\mathrm{C},p\right), \qquad (3)$$

where p is the pressure on the relevant density surface. The first term in Eqn. (3) is obtained from a call to gsw_enthalpy_CT25 and the last term via a call to gsw_enthalpy_SSO_0_CT25 which is simply a streamlined version of gsw_enthalpy_CT25.

In the call, gsw_geo_strf_McD_Klocker(SA,CT,p,gamma_n,p_gamma_n,'gn'), the user supplies the Absolute Salinity SA, Conservative Temperature CT and pressure down a vertical profile (or many such vertical profiles). The user requests the McDougall-Klocker geostrophic streamfunction on a series of density surfaces which intersect the successive vertical profiles at p_gamma_n. The last argument of the calling sequence to this function must be either 'gn' or 's2' which tells the code whether the interpolating variable for the reference cast is γ^{n} or σ_2.

Figure 1 below (from McDougall and Klocker (2010)) illustrates the errors in determining geostrophic velocities from various geostrophic streamfunctions in an ω-

surface. The surface is global in extent and had an average pressure of 1300 dbar. The McDougall-Klocker geostrophic streamfunction φ^{n} is labeled "Eqn (62)" in Figure 1(d).

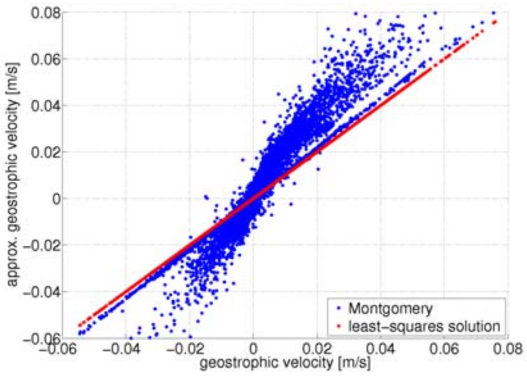

Figure 1(a)

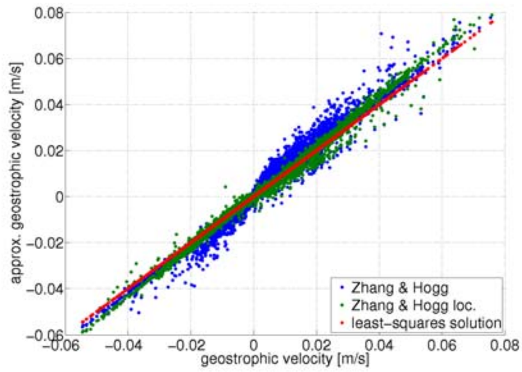

Figure 1(b)

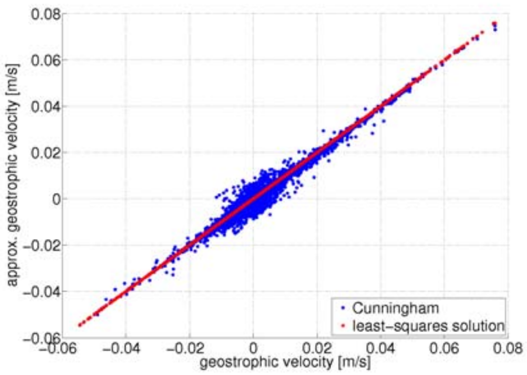

Figure 1(c)

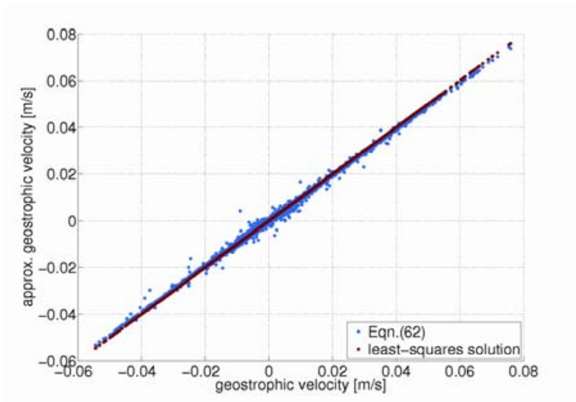

Figure 1(d)

Figure 1. Geostrophic velocities vs. approximate geostrophic velocities calculated from approximate geostrophic streamfunctions (all relative to the geostrophic velocity on the isobaric surface p = 2000 dbar) for (a) the Montgomery streamfunction π, (b) the Zhang and Hogg (1992) modification to the Montgomery streamfunction, $\pi^{\text{Z-H}}$, as well as the localized form of this streamfunction, $\tilde{\tilde{\pi}}^{\text{Z-H}}$, (c) the Cunningham streamfunction Π, and (d) the streamfunction φ^{n}. The geostrophic velocities are on an ω-surface with an average pressure of 1300 dbar.

References

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Klocker, A., T. J. McDougall and D. R. Jackett, 2009a: A new method for forming approximately neutral surfaces. *Ocean Sci.*, **5**, 155-172. http://www.ocean-sci.net/5/155/2009/os-5-155-2009.html

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McDougall T. J., D. R. Jackett, P. M. Barker, C. Roberts-Thomson, R. Feistel and R. W. Hallberg, 2010: A computationally efficient 25-term expression for the density of seawater in terms of Conservative Temperature, and related properties of seawater. To be submitted to *Ocean Science Discussions*.

Here follows section 3.30 from the TEOS-10 Manual (IOC *et al*. (2010)).

3.30 Geostrophic streamfunction in an approximately neutral surface

In order to evaluate a relatively accurate expression for the geostrophic streamfunction in an approximately neutral surface (such as an ω-surface of Klocker *et al.* (2009a,b) or a Neutral Density surface of Jackett and McDougall (1997)) a suitable reference seawater parcel $\left(\tilde{\tilde{S}}_{\mathrm{A}}, \tilde{\tilde{\Theta}}, \tilde{\tilde{p}}\right)$ is selected from the approximately neutral surface that one is considering, and the specific volume anomaly $\tilde{\tilde{\delta}}$ is defined as in (3.7.3) above. The approximate geostrophic streamfunction is given by (from McDougall and Klocker (2010))

$$\varphi^{\mathrm{n}}\left(S_{\mathrm{A}},\Theta,p\right) \;=\; \tfrac{1}{2}\left(P-\tilde{\tilde{P}}\right)\tilde{\tilde{\delta}}\left(S_{\mathrm{A}},\Theta,p\right) \;-\; \tfrac{1}{12}\rho^{-1}T_{\mathrm{b}}^{\Theta}\left(\Theta-\tilde{\tilde{\Theta}}\right)\left(P-\tilde{\tilde{P}}\right)^{2} \;-\; \int_{P_0}^{P}\tilde{\tilde{\delta}}\,dP' . \qquad (3.30.1)$$

This expression is very accurate when the variation of conservative temperature with pressure along the approximately neutral surface is either linear or quadratic. That is, in these situations $\nabla_{n}\varphi^{\mathrm{n}} \approx \tfrac{1}{\rho}\nabla_{z}P - \nabla\Phi_{0} = -\mathbf{k}\times\left(f\mathbf{v} - f\mathbf{v}_{0}\right)$ to a very good approximation. In Eqn. (3.30.1) $\rho^{-1}T_{\mathrm{b}}^{\Theta}$ is taken to be the constant value $2.7x10^{-15}\,\mathrm{K}^{-1}(\mathrm{Pa})^{-2}\mathrm{m}^{2}\mathrm{s}^{-2}$. This McDougall-Klocker geostrophic streamfunction is available from the GSW software as the function gsw_geo_strf_McD_Klocker.