

2.7 Specific volume

The specific volume of seawater v is given by the pressure derivative of the Gibbs function at constant Absolute Salinity S_A and *in situ* temperature t , that is

$$v = v(S_A, t, p) = g_P = \partial g / \partial P|_{S_A, T}. \quad (2.7.1)$$

Notice that specific volume is a function of Absolute Salinity S_A rather than of Reference Salinity S_R or Practical Salinity S_p . The importance of this point is discussed in section 2.8. When derivatives are taken with respect to *in situ* temperature, or at constant *in situ* temperature, the symbol t is avoided as it can be confused with the same symbol for time. Rather, we use T in place of t in the expressions for these derivatives.

For many theoretical and modeling purposes in oceanography it is convenient to regard the independent temperature variable to be potential temperature θ or Conservative Temperature Θ rather than *in situ* temperature t . We note here that the specific volume is equal to the pressure derivative of specific enthalpy at fixed Absolute Salinity when any one of η , θ or Θ is also held constant, as follows (from appendix A.11)

$$\partial h / \partial P|_{S_A, \eta} = \partial h / \partial P|_{S_A, \theta} = \partial h / \partial P|_{S_A, \Theta} = v. \quad (2.7.2)$$

The use of P in these equations emphasizes that it must be in Pa not dbar. Specific volume v has units of $\text{m}^3 \text{kg}^{-1}$ in both the SIA and GSW computer libraries.

2.8 Density

The density of seawater ρ is the reciprocal of the specific volume. It is given by the reciprocal of the pressure derivative of the Gibbs function at constant Absolute Salinity S_A and *in situ* temperature t , that is

$$\rho = \rho(S_A, t, p) = (g_P)^{-1} = \left(\partial g / \partial P|_{S_A, T} \right)^{-1}. \quad (2.8.1)$$

Notice that density is a function of Absolute Salinity S_A rather than of Reference Salinity S_R or Practical Salinity S_p . This is an extremely important point because Absolute Salinity S_A in units of g kg^{-1} is numerically greater than Practical Salinity by between 0.165 g kg^{-1} and 0.195 g kg^{-1} in the open ocean so that if Practical Salinity were inadvertently used as the salinity argument for the density algorithm, a significant density error of between 0.12 kg m^{-3} and 0.15 kg m^{-3} would result.

For many theoretical and modeling purposes in oceanography it is convenient to regard density to be a function of potential temperature θ or Conservative Temperature Θ rather than of *in situ* temperature t . That is, it is convenient to form the following two functional forms of density,

$$\rho = \tilde{\rho}(S_A, \theta, p) = \hat{\rho}(S_A, \Theta, p), \quad (2.8.2)$$

where θ and Θ are respectively potential temperature and Conservative Temperature, both referenced to $p_r = 0$ dbar. We will adopt the convention (see Table L.2 in appendix L) that when enthalpy h , specific volume v or density ρ are taken to be functions of potential temperature they attract an over-tilde as in \tilde{v} or $\tilde{\rho}$, and when they are taken to be functions of Conservative Temperature they attract a caret as in \hat{v} and $\hat{\rho}$. With this convention, expressions involving partial derivatives such as (2.7.2) can be written more compactly as (from appendix A.11)

$$\hat{h}_p = \tilde{h}_p = \hat{h}_p = v = \rho^{-1} \quad (2.8.3)$$

since the other variables are taken to be constant during the partial differentiation. Appendix P lists expressions for many thermodynamic variables in terms of the thermodynamic potentials

$$h = \hat{h}(S_A, \eta, p), h = \tilde{h}(S_A, \theta, p) \text{ and } h = \hat{h}(S_A, \Theta, p). \quad (2.8.4)$$

Density ρ has units of kg m^{-3} in both the SIA and GSW computer libraries.

Computationally efficient expressions for $\hat{\rho}(S_A, \Theta, p)$ and $\tilde{\rho}(S_A, \theta, p)$ involving 25 coefficients are available (McDougall *et al.* (2010b)) and are described in appendix A.30 and appendix K. These expressions can be integrated with respect to pressure to provide closed expressions for $\hat{h}(S_A, \Theta, p)$ and $\tilde{h}(S_A, \theta, p)$ (see Eqn. (A.30.6)).