3.10 Buoyancy frequency

The square of the buoyancy frequency (sometimes called the Brunt-Väisälä frequency) N^2 is given in terms of the vertical gradients of density and pressure, or in terms of the vertical gradients of potential temperature and Absolute Salinity (or in terms of the vertical gradients of Conservative Temperature and Absolute Salinity) by (the g on the left-hand side is the gravitational acceleration, and x, y and z are the spatial Cartesian coordinates)

$$\begin{aligned} g^{-1}N^2 &= -\rho^{-1}\rho_z + \kappa P_z \ = -\rho^{-1}\left(\rho_z - P_z / c^2\right) \\ &= \alpha^\theta \theta_z - \beta^\theta \, \partial S_\mathrm{A}/\partial z\big|_{x,y} \\ &= \alpha^\Theta \Theta_z - \beta^\Theta \, \partial S_\mathrm{A}/\partial z\big|_{x,y} . \end{aligned} \tag{3.10.1}$$

For two seawater parcels separated by a small distance Δz in the vertical, an equally accurate method of calculating the buoyancy frequency is to bring both seawater parcels adiabatically and without exchange of matter to the average pressure and to calculate the difference in density of the two parcels after this change in pressure. In this way the potential density of the two seawater parcels are being compared at the same pressure. This common procedure calculates the buoyancy frequency N according to

$$N^2 = -\frac{g}{\rho}\frac{\Delta\rho^\Theta}{\Delta z} = \frac{g^2\,\Delta\rho^\Theta}{\Delta P} , \tag{3.10.2}$$

where $\Delta\rho^\Theta$ is the difference between the potential densities of the two seawater parcels with the reference pressure being the average of the two original pressures of the seawater parcels. The last part of Eqn. (3.10.2) has used the hydrostatic relation $P_z = -g\rho$.