

Notes on the function gsw_sound_speed(SA,CT,p)

This function, **gsw_sound_speed**(SA,CT,p), evaluates the sound speed c of seawater from the 75-term polynomial expression for specific volume (as described in appendix A.30 and appendix K of the TEOS-10 Manual (IOC *et al.* (2010))).

Sound speed c is given in terms of the pressure derivative of *in situ* density $\hat{\rho}(S_A, \Theta, p)$ by (see sections 2.16 and 2.17 of the TEOS-10 Manual (IOC *et al.* (2010)))

$$c = \left(\frac{\partial \hat{\rho}}{\partial P} \Big|_{S_A, \Theta} \right)^{-\frac{1}{2}}. \quad (1)$$

Note that in order to have sound speed c in units of m s^{-1} the partial differentiation must be done with respect to pressure in Pa.

The sound speed evaluated from the 75-term polynomial expression for specific volume is discussed in Roquet *et al.* (2015) and in appendix A.30 and appendix K of the TEOS-10 Manual (IOC *et al.* (2010)). The sound speed evaluated from the 75-term polynomial of Eqn. (K.3) has an rms error over the “funnel” of 0.025 m s^{-1} which is a little less than the rms error of the underlying sound speed data that was incorporated into the Feistel (2008) Gibbs function, being 0.035 m s^{-1} (see rows 7 to 9 of Table O.1 of appendix O of the TEOS-10 Manual (IOC *et al.*, 2010)). Hence **gsw_sound_speed**(SA,CT,p) is almost as accurate as using the full TEOS-10 Gibbs function for evaluating sound speed in the ocean.

But for dynamical oceanography where α^Θ and β^Θ are the aspects of the equation of state that, together with spatial gradients of S_A and Θ , drive ocean currents and affect the calculation of the buoyancy frequency, we may take the 75-term polynomial expression for specific volume as essentially reflecting the full accuracy of TEOS-10. The fully accurate sound speed may be evaluated from either **gsw_sound_speed_CT_exact**(SA,CT,p) or **gsw_sound_speed_t_exact**(SA,t,p).

References

- IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from <http://www.TEOS-10.org>
- McDougall, T.J., D.R. Jackett, D.G. Wright and R. Feistel, 2003: Accurate and computationally efficient algorithms for potential temperature and density of seawater. *J. Atmosph. Ocean. Tech.*, **20**, pp. 730-741.
- Roquet, F., G. Madec, T. J. McDougall and P. M. Barker, 2015: Accurate polynomial expressions for the density and specific volume of seawater using the TEOS-10 standard. *Ocean Modelling*, **90**, pp. 29-43. <http://dx.doi.org/10.1016/j.ocemod.2015.04.002>

Here follows sections 2.16 and 2.17 of the TEOS-10 Manual (IOC *et al.* (2010)).

2.16 Isentropic and isohaline compressibility

When the entropy and Absolute Salinity are held constant while the pressure is changed, the isentropic and isohaline compressibility κ is obtained:

$$\begin{aligned}\kappa = \kappa(S_A, t, p) &= \rho^{-1} \frac{\partial \rho}{\partial P} \Big|_{S_A, \eta} = -v^{-1} \frac{\partial v}{\partial P} \Big|_{S_A, \eta} = \rho^{-1} \frac{\partial \rho}{\partial P} \Big|_{S_A, \theta} = \rho^{-1} \frac{\partial \rho}{\partial P} \Big|_{S_A, \Theta} \\ &= \frac{(g_{TP}^2 - g_{TT} g_{PP})}{g_P g_{TT}}.\end{aligned}\quad (2.16.1)$$

The isentropic and isohaline compressibility κ is sometimes called simply the isentropic compressibility (or sometimes the “adiabatic compressibility”), on the unstated understanding that there is also no transfer of salt during the isentropic or adiabatic change in pressure. The isentropic and isohaline compressibility of seawater κ produced by both the SIA and GSW software libraries (appendices M and N) has units of Pa^{-1} .

2.17 Sound speed

The speed of sound in seawater c is given by

$$c = c(S_A, t, p) = \left(\partial P / \partial \rho \Big|_{S_A, \eta} \right)^{0.5} = (\rho \kappa)^{-0.5} = g_P \left(g_{TT} / [g_{TP}^2 - g_{TT} g_{PP}] \right)^{0.5}. \quad (2.17.1)$$

Note that in these expressions in Eqn. (2.17.1), since sound speed is in m s^{-1} and density has units of kg m^{-3} it follows that the pressure of the partial derivatives must be in Pa and the isentropic compressibility κ must have units of Pa^{-1} . The sound speed c produced by both the SIA and the GSW software libraries (appendices M and N) has units of m s^{-1} .